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AN FEM ALGORITHM FOR FLOOD ROUTING

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CERTIFICATE

The thesis 'AN FEM ALGORITHM FOR FLOOD ROUTING' by Anil Kumar Gupta is hereby approved as a creditable report on research carried out and presented in a manner which warrants its acceptance as a prerequisite for the degree of MASTER OF TECHNOLOGY. The work has been carried out under my supervision and has not been submitted elsewhere for degree.

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ABSTRACT

Flood routing, forming one of the important facets of flood studies, has achieved vast progress in the last two decades due to general availability of digital computers. Amien's implicit method is one popular method in this category. The Finite Element Method (FEM) which is gaining popularity in various branches of mechanics has a great possibility of producing an effective numerical computation technique for handling a variety of flood routing problems. In this thesis an algorithm for flood routing in natural channels through the use of FEM with an implicit solution procedure is presented.

In this algorithm, one dimensional method of analysis is adopted and provision has been made for inclusion of lateral flow. The banded property of the global matrix has been used in finding a fast solution. The performance of the FEM algorithm is compared with that of Amien's implicit solution procedure. The computer CPU time, errors and stability aspects are studied. The FEM algorithm is found to give stable and convergent results at a very wide range of time steps.

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LIST OF SYMBOLS

A	=	Cross-sectional flow area
B, T	=	Top width
c	=	Celerity
c_u	=	Wave speed in upstream direction
c_d	=	Wave speed in downstream direction
e	=	Element
F, G, f	=	Functions
g	=	Gravitational acceleration
i	=	Distance node
j	=	Time node
k	=	Variables
L	=	Nodes on t-axis : Length of reach
N	=	Nodes on x-axis
n	=	Manning's n
P	=	Wetted Parameter
Q	=	Discharge
Q_n	=	Normal discharge
Q_b	=	Base flow
Qp	=	Peak flow

q	=	Lateral discharge per unit length
R	=	A/P; Residuals
S_o	=	Bed slope
S_f	=	Frictional slope
t	=	Time
u_x	=	Component of velocity in x-direction
V	=	Average velocity
x	=	Distance in the flow direction
y	=	Water depth
θ	=	Weighting factor
α	=	Time integration factor
ϕ	=	$1 - \theta$
β	=	Skewness factor
Δx	=	Distance step
Δt	=	Time step
$+$, u	=	Upstream
$-$, d	=	Down stream

LIST OF FIGURES

CHAPTER - I

INTRODUCTION

The importance of floods in human activities has generated a large number of studies towards understanding the various aspects of the flood flow. Floods are caused mainly by heavy rainfall in the catchment melting of snow or heavy releases at an upstream storage station. As the conveying channel may not be adequate to safely carry the incoming high discharges, some precautionary and other measures have to be taken in advance to prevent the lon of life and property. It requires the advance knowledge of the flood volume, its peak time, maximum stage to which it is likely to go, etc. at the selected points. This information may also be helpful for the regulation of different hydraulic structures in the system. This process by which we find out the channel outflows at certain selected stations of the channel, by knowing the upstream inflows in the channel is called flood routing. Inflows and outflows are in the form of stage or discharge hydrographs.

Stream flow is a phase of hydrologic cycle and is not well defined process. Actually it is a unification

of various physical processes. There is a continuous change from one state to another state with respect to space as well as time. For example parameters like channel geometry, bed roughness, stage etc. can get changed in space and time. Eventhough these can be modelled to a great extent, some assumptions and simplifications are necessary. One of the major simplification is that continuous processes are treated as discrete ones, wholly or to some extent. In these, we should have a check that our models and its various assumptions and simplifications do not have adverse effects on the resulting output.

Flood routing may be done by either a process approch or a system approach. If flood routing is done by using the process approach it is called hydraulic flood routing and by system approach, it is called hydrologic flood routing. The first one involve an mathematical modelling and the later one the conceptual modelling. These distinctions may be appropriate for well defined channels. In the case of natural channels, it may not be possible to have exact mathematical formulations and we have to apply conceptual approach at some places. Due to the remarkable development in the fields

of high speed digital computers and numerical methods, hydraulic models are playing a significantly large role in this field.

Unsteady flow in a rigid bed, open channel can be described by the St. Venant equations. These equations are the equations of conservation of mass and momentum and were first given by Saint Venant in 1871[1,5,8,15]. The St. Venant equations are two non-linear, first order, first degree partial differential equations. There are no mathematical solution to them and they can be solved numerically for specified boundary and initial conditions.

The solution procedures of St. Venant equations are discusses in Chapter II. Chapter III discuss briefly the commonly used finite difference method (FDM) and its limitations. An FEM algorithm is developed and its characteristics enumerated in Chapter IV. A comparative study of the FDM and the FEM algorithm is presented with the help of two examples in Chapter V. The significant conclusions of this study are collated in Chapter VI. Appendix - A and B presents the listing of the FDM and FEM algorithms implemented through FORTRAN-10 in the DEC-1090 system at Indian Institute of Technology, Kanpur.

CHAPTER - II

NUMERICAL SOLUTION OF THE SAINT-VENANT EQUATIONS

2.1 BASIC EQUATIONS

The St. Venant equations for unsteady flow are given as[1]:

$$V\frac{\partial A}{\partial x} + A\frac{\partial V}{\partial x} + \frac{\partial A}{\partial t} - q = 0 \qquad (2.1)$$

and
$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} - g(S_o - S_f) + \frac{(V - u_x)q}{A} = 0 \qquad (2.2)$$

where V = average velocity

A = cross-sectional flow area

x = distance in the flow direction

y = water depth

t = time

S_o = bed slope

S_f = friction slope

q = lateral incoming discharge per unit length

u_x = average component of inflow velocity in x direction

g = gravitational acceleration.

The first equation is the continuity equation and the second one is the momentum equation. Derivation of these equations assumes that[18]:

(i) the flow is one dimensional

(ii) the fluid is homogeneous and incompressible

(iii) the channel bed is fixed

(iv) the channel alignment is approximated to be straight line

(v) the bottom slope is small

(vi) the flow is gradually varied

(vii) the water surface is horizontal across a cross section

(viii) the velocity is constant across a cross section

(ix) the wind resistance is neglected.

2.2 SOLUTION OF ST.VENANT EQUATIONS

The various methods of solving St. Venant equations can be broadly classified as (a) approximate numerical method, (b) complete numerical method. The approximate methods are based on drastically curtailed equation of motion. The complete numerical method obtains

the solution by St. Venant equations. The Direct Method, Finite Element Method and Method of Characteristics (Rectangular grid and Characteristics nodes) belong to this category. These are further classified in the implicit and explicit method. Subramanya[16] has given general description of direct method and of method of characteristics. Different finite differencing schemes are available for these two methods[1,2,10,13,15]. Finite Element Method (FEM) is of recent origin and is in developing stage. Probably Cooley and Moin[6] are the first one to propose an algorithm for FEM to solve St. Venant equations.

2.3 INITIAL CONDITIONS

To start the computation, the initial values of unknowns are required. These are discharges and water depths at all the nodal points along the river. Generally one assumes that the flow to be steady before the start of flood wave. Hence initial values may be computed by the solution of gradually varied steady flow equations.

2.4 BOUNDARY CONDITIONS

Boundary conditions are must for a solution of a system in space and time. Liggett and Woolhiser[10]

specified that one condition at the upstream is needed if the flow is subcritical and two conditions are needed at the upstream if the flow is supercritical. In addition for subcritical flow one condition at the down stream boundary also is needed. These may be in the form of specified depth or velocity at a time. If the flow changes from subcritical to supercritical and vice versa, the boundary specifications may get changed.

The upstream boundary condition is generally a discharge or stage hydrograph. The down stream boundary condition is given by a rating curve. It carries the assumption that this section is not subject to back-water effects from downstream regulation. However if some flow restriction exist the computation should either be carried out with the downstream boundary located at this restriction, or a rating curve obtained from back water computations may be used.

2.5 DYNAMIC AND KINEMATIC MODELS

Dynamic waves propogate in two systems of characteristics; in the upstream and downstream direction. St. Venant equations describe the dynamic waves. Upstream

travel speed is given by

$$C_u = V - \sqrt{gy} \tag{2.3}$$

and downstream travel speed is given by

$$C_d = V + \sqrt{gy} \tag{2.4}$$

Kinematic waves posses only one system of characteristics. Their theoretical speed can be given by Kleitz-Seddon law[18].

$$C = \frac{dQ}{dA} \tag{2.5}$$

In a natural flood wave both types of wave movement are present. The bulk of flood wave moves substantially as a kinematic wave, while the dynamic wave fronts move in front and behind the main body of the flood wave. Observations supports the theory of kinematic waves, but the flood wave does not steepen as much as predicted by it. Henderson[8] showed that the magnitude of the deviations from kinematic wave behaviour depends mainly on the ratio between bed slope and wave slope and on Froude Number.

For dynamic wave the waiting curve is a looped one and is given by

$$Q = Q_n \sqrt{1 - \frac{1}{S_o}\frac{\partial y}{\partial x} - \frac{V}{S_o g}\frac{\partial V}{\partial x} - \frac{1}{S_o g}\frac{\partial V}{\partial t}} \qquad (2.6)$$

where Q_n is the normal discharge and can be given by Chezy's, Manning's or any other emperical resistance equations[16].

For kinematic wave the discharge is equal to normal discharge and the rating curve, is a linear curve is given by

$$Q = Q_n \qquad \ldots \qquad \ldots \qquad (2.7)$$

The models based on kinematic waves are called Kinematic or Approximate models, while the models based on Dynamic waves are called Dynamic or Complete models[16,18].

2.6 CHANNEL CHARACTERISTICS

Channel geometry may be described in various forms. Weinmann[18] has distinguished four broad ways.

(i) Replacement of actual river by a uniform channel for a total length of reach.

(ii) Replacement of actual river by a series of prismatic channels of different width.

(iii) Direct use of polygonal section.

(iv) Stochastic generation of cross-section.

Resistance properites are usually based on Chezy's or Manning's formula. The relevant resistance coefficients may be taken either constant or as a function of position and depth.

2.7 ROUTING PARAMETERS

These consist of the time and space increments and the different weighting coefficients. Variation of these parameters may affect the efficiency and accuracy of the numerical procedure adopted and hence the proper selection is of these great importance.

CHAPTER - III

IMPLICIT FINITE DIFFERENCE METHOD

3.1 INTRODUCTION

Among the direct method and method of characteristics, the finite difference method using an Implicit numerical procedure has been generally accepted as the most convenient method. Among the various implicit finite differencing schemes, Ameins Implicit Scheme[1] is considered to be the most efficient one[13]. In this Chapter an Implicit FDM Algorithm which is a modification of the Amien's work[1,2] is developed. This is used as a basis for comparison of the efficiency of FEM algorithm presented in the subsequent chapter of this thesis.

3.2 DETAILED DESCRIPTION

The basic equations are taken in the form used by Amein and Fang in 1970[1] as

$$\frac{\partial y}{\partial t} + \frac{A}{B}\frac{\partial V}{\partial x} + V\frac{\partial y}{\partial x} - \frac{q}{B} = 0 \qquad (3.1)$$

and $$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial V}{\partial x} - g\,(S_o - S_f) + \frac{Vq}{A} = 0 \qquad (3.2)$$

where $B = \frac{dA}{dy}$

Here u_x has been assumed zero.

The equations (3.1) and (3.2) are replaced by the algebric finite difference equations and then the solution of these equations will be obtained. To find the numerical solution of equations (3.1) and (3.2) by implicit method the (x, t) plane is discretized in a rectangular net covering the whole river reach and the time for which computations are to be carried. The x axis will be

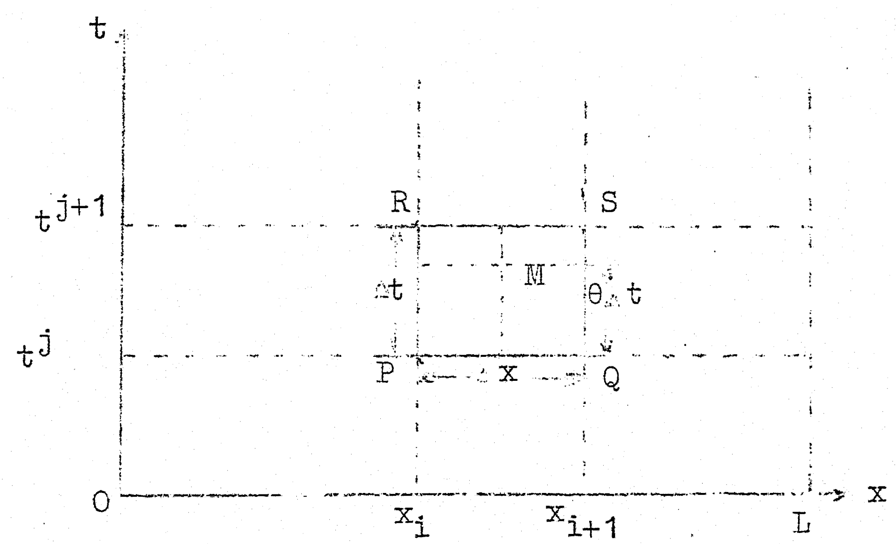

FIG.3.1 (x,t) PLANE IN AMEIN'S IMPLICIT SCHEME

be representing the initial condition and the line paralled to it will denote times. (Fig. 3.1). The spacing between these lines will be Δx. Similarly Δt is the time increment and t axis denotes the

upstream boundary. The line, parallel to it, drawn at $x = L$, where L is the length of the reach will denote downstream boundary. Line in between will represent different nodal stations (Fig. 3.1). The forming grid will be double subscripted one. The values of Δx and Δt need not to be constant. But however these has been taken constant in the present study.

Assuming that all the variables are known at all nodes on the row having $t = t^j$ (PQ) and variables at the row having $t = t^{j+1}$ (RS) are unknown where $t^{j+1} = t^j + \Delta t$. In a four point method the values of any function and its derivatives with respect to time and space are expressed in terms of the values of function at the four grid points. If the grid is found by the state lines $t = t^j$, $t = t^{j+1}$, $x = x_i$ and $x = x_{i+1}$ (PQ, RS, PR and QS in Fig. 3.1) and it is assumed that a point M (Fig. 3.1) is centered between the space grid line and positioned at $t = t^j + \theta(t^{j+1} - t^j)$ where θ is a weighting factor, then the function ( ) and its derivatives will be

$$k = \frac{\theta(k_i^{j+1} + k_{i+1}^{j+1}) + (1 - \theta)(k_i^j + k_{i+1}^j)}{2} \qquad (3.3)$$

$$\frac{\partial k}{\partial x} = \frac{\theta(k_{i+1}^{j+1} - k_i^{j+1}) + (1-\theta)(k_{i+1}^{j} - k_i^{j})}{\triangle x_i} \quad (3.4)$$

and

$$\frac{\partial k}{\partial t} = \frac{k_i^{j+1} + k_{i+1}^{j} - k_i^{j} - k_{i+1}^{j}}{2 \triangle t_j} \quad (3.5)$$

The value of θ may be assumed to be between 0.5 and 1.0. For a value of 1 the scheme is known as the fully implicit scheme and for a value of 0.5, it is known as the box scheme. Amain and Fang in 1970[1] have used box scheme and Amein and Chu in 1975[2] have used the fully implicit scheme. As per Quinn and Wylie[14] implicit schemes are weakly stable for θ in between p0.5 and 0.6. Chaudri and contractor [4] have shown that the implicit values are more accurate but generally unstable for a value close to 0.5. Converse is the case for a value of $\theta = 1.0$. Weinmann[18] took the value of $\theta = 1$. However in the present study various values of θ between 0.5 and 1.0 have been tested to know the sensitivity of the scheme and to suggest the optimum value of θ for use.

By putting values of functions and variables i.e. y, V, $\frac{\partial y}{\partial t}$, $\frac{\partial y}{\partial x}$ etc. obtained from equations 3.3, 3.4

and 3.5 in equations 3.1 and 3.2, finite difference form of these equations are obtained. In this friction slope S_{fi}^{j} at a point (i, j) is computed by Manning's formula

$$S_{fi}^{j} = \frac{(n_i^j)^2 \; V_i^j \; |V_i^j|}{(R_i^j)^{4/3}} \qquad (3.6)$$

where $R_i^j = A_i^j / P_i^j$ (hydraulic radius)

and P_i^j = wetted parameter

Hereafter time dependent variable will not carry any superscript, if time line is j+1.

Finite Difference form are given as

$$F_i\,(y_i,\; V_i,\; y_{i+1},\; V_{i+1}) \quad a + a' + \frac{\Delta t}{4\Delta x} \cdot$$

$$((c + c')(b+b') + (h+h') + (e + e')) + 0.5 \quad t\; q$$

$$(w + w') = 0 \qquad (3.7)$$

and $G_i(y_i,\; V_i,\; y_{i+1},\; V_{i+1})$

$$= (a'' + a''' + \frac{\Delta x}{g\Delta t}(b''+b''') + \frac{1}{4g}(c''+c''')$$

$$(d''+d''') + \frac{\Delta x}{2}(h''+h''') + \frac{\Delta x}{2g}\, q(o''+o''') = 0$$

$$(3.8)$$

where $a = y_{i+1} + y_i$

$$b = 2\theta(V_i + V_{i+1})$$

$$c = 2\theta\,(y_{i+1} - y_i)$$

$$e = 2\theta\,(V_{i+1} - V_i)$$

$$h = 2\theta\,\left(\frac{A_i}{B_i} + \frac{A_{i+1}}{B_{i+1}} \right)$$

$$w = 2\theta\,\left(\frac{1}{B_i} + \frac{1}{B_{i+1}} \right)$$

$$a' = y^j_{i+1} - y^j_i$$

$$b' = 2\phi\,(V^j_i + V^j_{i+1}) \qquad (3.9)$$

$$c' = 2\phi\,(y^j_{i+1} - y^j_i)$$

$$e' = 2\phi\,(V^j_{i+1} - V^j_i)$$

$$h' = 2\phi\,\left(\frac{A^j_i}{B^j_i} + \frac{A^j_{i+1}}{B_{i+1}} \right.$$

$$a'' = c$$

$$b'' = V_i + V_{i+1}$$

$$c'' = b$$

$$d'' = e$$

$$h'' = 2\theta(S_{fi} + S_{f\ i+1})$$

$$o'' = 2\theta \ \left(\frac{V_j}{A_i} + \frac{V_{i+1}}{A_{i+1}} \right)$$

$$a''' = c'$$

$$b''' = -V_i^j - V_{i+1}^j \qquad (3.9)$$

$$c''' = b'$$

$$d''' = e'$$

$$h''' = 2\emptyset(S_{fi}^j + S_{f\ i+1}^j) - 4\ S_o$$

$$o''' = 2\emptyset \ \left(\frac{V_i^j}{A_i^j} + \frac{V_{i+1}^j}{A_{i+1}^j} \right)$$

where $\emptyset = 1 - \theta$

It amy be seen that the constants given in Eq. 3.9 carrying a (''') are independent of the variables at time (j+1). Similarly unscripted constants and the constants carrying ('') are independent of the variable at time j. This property should be kept in mind to avoid duplicate computations of these constants.

In equations 3.7, 3.8, and 3.9 only the variables with the superscript j+1 are the unknowns. These are

y_i, V_i, y_{i+1}, V_i. (N-1) interior points provide 2(N-1) equations for the 2N unknowns. Two additional equations are available from the boundary conditions.

If the stage hydograph at up stream boundary is given then

$$G_o\ (y_1) = y_1 - f_1(t^{j+1}) = 0 \tag{3.10}$$

and if discharge hydrograph is given then

$$G_o\ (y_1,\ V_1) = V_1\ A_1 - f_1(t^{j+1}) = 0 \tag{3.11}$$

where $f_1(t^{j+1})$ is stage or discharge value taken from the respective hydrograph as the case may be.

Similarly one equation will be obtained from down stream boundary condition. This equation will be

$$F_N\ (y_N,\ V_N) = V_N\ A_N - Q \tag{3.12}$$

where Q will be calculated from equations 2.6 or 2.7 for dynamic and kinematic cases respectively. In the present study Normal flow, Q_n, is calculated by using Manning's equations.

Now our equations constitute a set of 2N nonlinear algebric equations in 2N unknowns. The equations can be

assembled as follows

$$
\begin{aligned}
R_1 &\equiv G_o\,(y_1,\ V_1) && = 0 \\
R_{2i} &\equiv F_i(y_i,\ V_i,\ y_{i+1},\ V_{i+1}) && = 0 \\
& \qquad \text{for } i = 1 \text{ to } N-1 && \\
R_{(2i+1)} &\equiv G_i(y_i,\ V_i,\ y_{i+1},\ V_{i+1}) && = 0 \\
& \qquad \text{for } i = 1 \text{ to } N-1 && \\
R_{2n} &\equiv F_N\,(y_N,\ V_N) && = 0
\end{aligned}
\tag{3.13}
$$

Although Eq. 3.13 carries 2N unknowns yet each equation contains a maximum of four unknowns and the whole matrix can come in banded matrix of width five. The generalized Newton iteration method is used herein to find the solution of this system. Relating residuals of Eq. 3.13 to the partial derivatives, as per Newton iteration method(1)

$$\frac{\partial G_o}{\partial y_1}\,d\,y_1 + \frac{\partial G_o}{\partial V_1}\,dV_1 = R_1$$

$$\frac{\partial F_i}{\partial y_i}\,d_{yi} + \frac{\partial F_i}{\partial V_i}\,dV_i + \frac{\partial F_i}{\partial y_{i+1}}\,dy_{i+1} + \frac{\partial F_i}{\partial V_{i+1}}\,dV_{i+1} = R_{2i}$$

for $i = 1$ to $N-1$

$$\frac{\partial F_N}{\partial y_N} dy_N + \frac{\partial F_N}{\partial V_N} dV_N = R_{2N} \qquad (3.14)$$

Which constitute a set of 2N linear equations in 2N unknowns and in which dy_i and dV_i for i having value from 1 to N are the additive corrections to be applied to the respective y_i and V_i values at the (j+1) time. Knowing the values of dy_i and dV_i from the solution of above given set of equations the values of y_i and V_i may be modified in this manner. The method used here is Gaussian elemination method using banded matrix technique. At the terminal iteration, when the values of dy_i and/or dV_i are less then the permissible errors, we take the values of variables found as final and advance for time step (j+2).

The expression for the partial derivatives are obtained by differentiating Eq. 3.7 and 3.8 partially with respect to the variables

$$\frac{\partial F_i}{\partial y_i} = 1 - \frac{\Delta t}{2\Delta x}((b + b') + ((e + e')$$

$$\theta(1 - \frac{A_i}{B_i^2} \cdot (\frac{\partial B}{\partial y})_{i+1}))) - \frac{q\Delta t}{B_i^2} \cdot (\frac{\partial B}{\partial y})_i \theta \qquad (3.15)$$

$$\frac{\partial F_i}{\partial y_{i+1}} = 1 + \frac{\Delta t}{2 \Delta x} \left((b + b')+((e + e') \right.$$

$$\left. \theta\left(1 - \frac{A_{i+1}}{(B_{i+1})^2} \left(\frac{\partial B}{\partial y}\right)_{i+1}\right)\right)\Bigg) - \frac{q \Delta t}{(B_{i+1})^2} \left(\frac{\partial B}{\partial y}\right)_{i+1} \theta \tag{3.16}$$

$$\frac{\partial F_i}{\partial V_i} = \frac{\Delta t}{4 \Delta x} (c + c' - h - h') \tag{3.17}$$

$$\frac{\partial F_i}{\partial V_{i+1}} = \frac{\Delta t}{4 \quad x} (c + c' + h + h') \tag{3.18}$$

$$\frac{\partial G_i}{\partial y_i} = 2\theta \left(-1 + \frac{2}{3} \Delta x \; S_{fi} \left(\frac{1}{P_i} \left(\frac{\partial P}{\partial y}\right)_i - \frac{B_i}{A_i^{j+1}} \right)\right.$$

$$\left. - \frac{q \Delta x}{2g} V_i \frac{B_i}{(A_i)^2} \right) \tag{3.19}$$

$$\frac{\partial G_i}{\partial y_{i+1}} = 2\theta \left(1 + \frac{2}{3} \Delta x \; S_{fi+1} \left(\frac{1}{P_{i+1}} \left(\frac{\partial P}{\partial y}\right)_{i+1} - \frac{B_{i+1}}{A_{i+1}} \right)\right.$$

$$\left. - q \frac{\Delta x}{2g} V_{i+1} \frac{B_{i+1}}{(A_{i+1})^2} \right) \tag{3.20}$$

$$\frac{\partial G_i}{\partial V_i} = \frac{1}{g} \left(\frac{\Delta x}{\Delta t} \right) + \frac{\theta}{2g} (d''' - c''' - 4\theta V_i^2)$$

$$+ 2\theta \Delta x \frac{S_{fi}}{V_i} + \frac{\theta q \Delta x}{g A_i} V_i \tag{3.21}$$

$$\frac{\partial G_i}{\partial V_{i+1}} = \frac{1}{g} \frac{\Delta x}{\Delta t} + \frac{\theta}{2g} (c''' + d''' + 4\theta V_{i+1}^2)$$

$$+ 2\theta \Delta x \frac{S_{f\ i+1}}{V_{i+1}} + \frac{\theta \quad q \Delta x}{g \ A_{i+1}} V_{i+1} \qquad (3.22)$$

Similiarly partial derivatives has been found for boundaries. If the stage hydrograph is given at upstream the partial derivatives will be

$$\frac{\partial G_o}{\partial y_1} = 1 \qquad (3.23)$$

$$\frac{\partial G_o}{\partial V_1} = 0 \qquad (3.24)$$

and if the discharge hydrograph is given at the upstream the partial derivatives will be

$$\frac{\partial G_o}{\partial y_1} = - V_1 \quad B_1 \qquad (3.25)$$

$$\frac{\partial G_o}{\partial V_1} = - A_1 \qquad (3.26)$$

Similarly for Kinematic wave condition at down stream boundary the values will be given by

$$\frac{\partial F_N}{\partial y_N} = V_N\ B_N + \frac{1}{n_N}\ \frac{2}{3}\ R_N^{2/3}\ (R_N\ (\frac{\partial P}{\partial y})_N^{5/3} - 2.5\ B_N)\ S_o^{1/2} \tag{3.27}$$

$$\frac{\partial F_N}{\partial V_N} = A_N \tag{3.28}$$

and for dynamic wave condition the down stream boundary these values will be given by

$$\frac{\partial F_N}{\partial y_N} = V_N\ B_N + \frac{1}{n_N}\ \frac{2}{3}\ R_N^{2/3}\ (R_N\ (\frac{\partial P}{\partial y})_N^{5/3} - 2.5\ B_N)\ S_o^{1/2}\ f_{Q_N} + 0.5\ \frac{Q_{nN}}{f_{Q_N}\cdot S_o \triangle x} \tag{3.29}$$

and $$\frac{\partial F_N}{\partial V_N} = A_N + 0.5\ \frac{Q_{nN}}{f_{Q_N}\cdot S_o \triangle xg}(2V_N - V_{N-1} + \frac{\triangle x}{\triangle t}) \tag{3.30}$$

where

$$f_{Q_N} = (1 - \frac{1}{S_o \triangle x}\ (y_N - y_{N-1}) - \frac{1}{S_o\ g \triangle x}\ V_N(V_N - V_{N-1}) - \frac{1}{S_o g \triangle t}\ (V_N - V_N^{\,j}))^{1/2} \tag{3.31}$$

$$Q_{nN} = \frac{1}{n_N}\ A_N\ R_N^{2/3}\ S_o^{1/2} \tag{3.32}$$

and n_N = Mannings' roughness constant

3.3 PROGRAMMING

The above implicit method of numerical solution was programmed in FORTRAN-10 and implemented at DEC-1090 at Indian Institute of Technology, Kanpur. Initial condition, type of boundary condition, No. of nodes on x and t axis, time increament, space increment, lateral flow, bed slope and the permissible termination errors are being read as the input data. However input has also to be given for the form of output needed viz. the stations where the output is needed and the time increament after which out put is needed. Provision has been made for giving stage and discharge hydrograph. If the programme does not iterate to a desired level in a specified number of iterations, it is mentioned in the output and the programme is terminated. The variable viz. Area, Breadth, Parameter, Manning's n who are time and space dependent are to be defined in the form of subrouting functions. There derivatives are also introduced in the same way. Upstream hydrograph, either stage on discharge, is given in a functional subroutine as a function of time. Lateral flow is available by subroutine which gives the discharge and their velocity component in the direction of flow at a particular time for all the lateral flows.

Eqs. 3.13 and from 3.15 to 3.22 has been given in the form of functions. All the details has been provided in the main body of the programme. Flow chart of the programme is shown in Fig. 3.2. A listing of the FORTRAN Program (FDM) is included in Appendix - A.

3.4 CHARACTERISTICS OF FDM PROGRAM

The size of distance steps does not have much influence on the solution accuracy, as long as the variation of the flow characterises along the channel can be appropriately represented.[18] In the present study selection of distance steps has been done by the consideration of stability. In the method of characteristics selection of Δt is done by following the Courand criteria ($\frac{\Delta t}{\Delta x} |V \pm c| \leq 1$). However in the present study even greater ($\approx$ 2.5 times Courant value) values produced an stable result. The θ should be selected for giving accurate, stable and convergent results. In the present study it has been found that $\theta = 0.6$ satisfies the above properties to a great extent and consume less CPU time. However $\theta = 0.55 - 0.65$ gives acceptable results and it may be selected in a way that it reproduces the desired hydrograph.

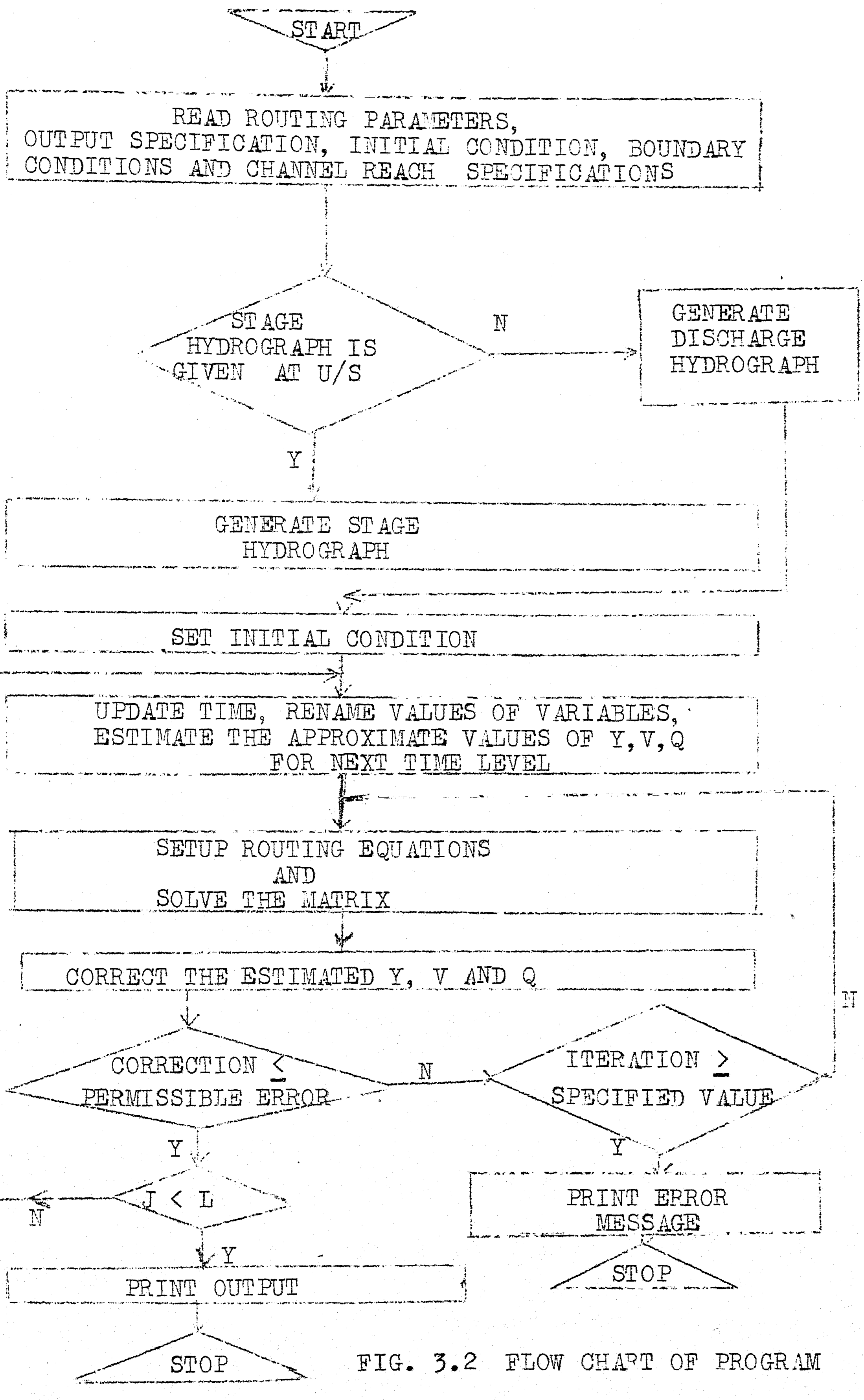

FIG. 3.2 FLOW CHART OF PROGRAM

In the present study verification has been done by the examples worked out by others and comparison of results is done among themselves and with them. If two or more sets of parameters gives practically identical results, one consuming less time is considered the best one.
It has been observed that the results generally differ at the peak. The resultant hydrograph reproducing peak values, peak time and shape has been concluded the best.
In this study the solution is taken as convergent, if it converges in 40 iteration steps.

CHAPTER - IV

IMPLICIT FINITE ELEMENT METHOD

4.1 INTRODUCTION

In finite element one discretize the in elements and try for solution. The details of the FEM can be seen in references[7,11,12,20]. Gooley and Moin in 1976[6] gave an FEM solution to the flood routing problems. They took linear elements and used predictor corrector method. and irregular time steps which were less at peaks and more else where were used. King in 1977[9] give another solution to the flood routing problem using FEM. He used time integration scheme instead of predictor corrector method.

4.2 DETAILED DESCRIPTION

The absic equations were taken in a different form here. First and second terms of Eq. 2.1 are grouped together giving new form of continuity equation

$$B \frac{\partial y}{\partial t} + \frac{\partial Q}{\partial x} - q = 0 \tag{4.1}$$

Eq. 2.2 is multiplied by A and Eq. 4.1 by V and by adding the results we obtain modified form of momentum equation.

$$\frac{\partial Q}{\partial t} + \frac{\partial (VQ)}{\partial x} + gA \frac{\partial y}{\partial x} - gA(S_o - S_f) - u_x q = 0 \qquad (4.2)$$

Equations 4.1 and 4.2 have been used in the finite element formulation.

The system is discretized as used by Cooley and Moin(6) into linear elements in space for a specified time. Hence finite element solution is employed only for space variables. King(9) has suggested the use of finite difference time integration scheme for finding the values of partial derivatives of variables y and Q with respect to time. First describing y as a function of time at a particular position in space,

$$y = y_1 + a\,t + b\,t^{\alpha} \qquad (4.3)$$

then $$\frac{\partial y}{\partial t} = a + \alpha\, b\, t^{\alpha-1} \qquad (4.4)$$

Putting the value of $t^{\alpha-1}$ from Eq. 4.4 in Eq. 4.3 we get

$$\frac{\partial y}{\partial t} = a + \frac{\alpha}{t}(y - y_1) - \alpha\, a$$

at $t = 0$; $\frac{\partial y}{\partial t_1} = a$

then at time Δt

$$\left(\frac{\partial y}{\partial t}\right)_2 = \frac{\alpha}{\Delta t}(y_2 - y_1) + (1 - \alpha)\left(\frac{\partial y}{\partial t}\right)_1 \qquad (4.5)$$

Here subscript 2 denote the variables at time Δ t. Similarly

$$\left(\frac{\partial Q}{\partial t}\right)_2 = \frac{\alpha}{\Delta t}(Q_2 - Q_1) + (1 - \alpha)\left(\frac{\partial Q}{\partial t}\right)_1 \qquad (4.6)$$

For $\alpha = 1$ this scheme reduces to the linear integrration scheme and for $\alpha = 2$ the scheme is quadratic integration scheme. King[9] used the value of $\alpha = 1.5$. However in the present study different values of α have been tested.

A formulation based on the method of weighted residuals has been used herein to develop the finite element equations. The basic Eqs. 4.1 and 4.2 are written in the integral form and the shape functions of the finite element approximation are used as the weighting parameters. As per Galkerine procedure,

$$\sum_e \int_{L_e} N_i^{(e)} \left(\frac{\partial Q}{\partial t} + V \frac{\partial Q}{\partial x} + Q \frac{\partial V}{\partial x} + gA \frac{\partial y}{\partial x} - 2A\,(S_o - S_f - u_x\, q)\right) dx = 0 \qquad (4.7)$$

and

$$\sum_e \int_{L_e} N_i^{(e)} \left(B \frac{\partial y}{\partial t} + \frac{\partial Q}{\partial x} - q\right) dx = 0 \qquad (4.8)$$

in which L_e is the river reach in the element e and $N_i^{(e)}$

are coordinate functions for the node i in the element e.

The variables Q, y, A, B and S_f are assumed to be linearly variable with respect to x in each discrete element, e.g.

$$y = N_i \; y_i + N_{i+1} \; y_{i+1} \qquad (4.9)$$

In which coordinate function are given as

$$N_i = \frac{x_{i+1} - x}{x_{i+1} - x_i}$$

and (4.10)

$$N_{i+1} = \frac{x - x_i}{x_{i+1} - x_i}$$

where i is the node bounding the element withing which the function is approximated. As variables are assumed linear in the element under consider the nodal values of elements not bounded by the node will have no effect on the values of the functions and hence $N_i^{(e)}$ will be zero for all elements not bounded by element e.

The terms of Eqns. 4.7 and 4.8 are differentiated and integraded as per the indication after putting the

different approximating functions. The equations for interior nodes will be

$$\delta_{i-(1/2)} \frac{\partial Q_{i-1}}{\partial t} + 2\,(\delta_{i-(1/2)} + \delta_{i+(1/2)} \frac{\partial Q_i}{\partial t}$$

$$+ \delta_{i+(1/2)} \frac{\partial Q_{i+1}}{\partial t} + V_{i-(1/3)} (Q_i - Q_{i-1})$$

$$+ V_{i+(1/3)} (Q_{i+1} - Q_i) + Q_{i-(1/3)}(V_i - V_{i-1})$$

$$+ Q_{i+(1/3)} (V_{i+1} - V_i) + g\,(A_{i-(1/3)}(y_i - y_{i-1})$$

$$+ A_{i+(1/3)} (y_{i+1} - y_i)) \quad -g(\delta_{i-(1/2)} A_{i-(1/3)} S_{ou}$$

$$+ \delta_{i+(1/2)} A_{i+(1/3)} \; S_{od} \,)$$

$$+ \frac{1}{2}\, g\,(\delta_{i-(1/2)} A_{i-(1/2)} \; n_u^2 \, F_{f\ i-1}$$

$$+ \;(\delta_{i-(1/2)} A_{i-(1/4)} \, n_u^2 + \delta_{i+(1/2)} A_{i+(1/4)} \; n_d^2 \,)$$

$$F_{fi} + \delta_{i+(1/2)} A_{i+(1/2)} \, n_d^2 \; F_{f\ i+1} \,)$$

$$-3(\delta_{i-(1/2)} \; u_{xu} \; q_u + \delta_{i+(1/2)} u_{xd} \; q_d) = 0 \qquad (4.11)$$

and

$$\delta_{i-(1/2)} \; T_{i-(1/2)} \; \frac{\partial y_{i-1}}{\partial t} + \delta_{i+(1/2)} \; T_{i+(1/4)}$$

$$\frac{\partial y_i}{\partial t} + \delta_{i+(1/2)} \; T_{i+(1/2)} \; \frac{\partial y_{i+1}}{\partial t} \; 6(Q_{i+1} - Q_{i-1})$$

$$- \; 6(\delta_{i-(1/2)} \; q_u + \delta_{i+(1/2)} \; q_d) = 0 \qquad (4.12)$$

where $\delta_{i\pm(1/2)} = \pm (x_{i\pm 1} - x_i)$

$$A_{i\pm(1/2)} = A_i + A_{i \pm 1}$$

$$A_{i\pm(1/3)} = 2A_i + A_{i\pm 1}$$

$$A_{i\pm(1/4)} = 3A_i + A_{i\pm 1}$$

$$F_f = \frac{S_f}{n^2}$$

$$V = Q/A \qquad (4.13)$$

V, Q, T terms at fractional nodal distance are defined in the same way as A has been defined. The subscript u and d indicate the upstream and downstream elements with respect to node i.

By this way we get (2 N-4) equations in 2N unknowns. Two more equations are get by treating the Eq. 4.8 in the same way for first and the last elements

$$\delta_{1+(1/2)} \left(T_{1+(1/4)} \frac{\partial y_1}{\partial t} + T_{1+(1/2)} \frac{\partial y_2}{\partial t}\right)$$

$$+ 6(Q_2 - Q_1 - \delta_{1-(1/2)} q_d) = 0 \qquad (4.14)$$

and

$$\delta_{N-(1/2)} \left(T_{N-(1/4)} \frac{\partial y_N}{\partial t} + T_{N-(1/2)} \frac{\partial y_{N-1}}{\partial t}\right)$$

$$+ 6 (Q_N - Q_{N-1} - \delta_{N-(1/2)} q_u) = 0 \qquad (4.15)$$

The remaining two equations needed for solution may be obtained by the boundary conditions. If the upstream stage hydrograph is given then the upstream boundary condition will be

$$y_1 - f_1 (t^{j+1}) = 0 \qquad (4.16)$$

and if discharge hydrograph is given then

$$Q_1 - f_1 (t^{j+1}) = 0 \qquad (4.17)$$

where $f_i(t^{j+1})$ carries the same meaning as given in Finite difference method. The downstream boundary

condition will be

$$Q_N - Q = 0 \qquad \ldots \qquad \ldots \qquad (4.18)$$

where Q is given by Eqs. 2.6 or 2.7 for dynamic and Kinematic cases respectively. In the present study Normal flow, Q_n is calculated by using Manning's equation.

Now our equations constitute a set of 2N nonlinear algebric equations in 2N unknowns, namely $\frac{\partial y_i}{\partial t}$, $\frac{\partial Q_i}{\partial t}$, for i values of 1 to N. However the $\frac{\partial y_i}{\partial t}$ and $\frac{\partial Q_i}{\partial t}$ can also be furthet written in the y_i and Q_i as per Eqs. 4.5 and 4.6 respectively. The equations can be assembled as follows:

$$\begin{aligned}
R_1 &\equiv F_1 (y_1, Q_1) &= 0 \\
R_2 &\equiv G_1(y_1, Q_1, y_2, Q_2) &= 0 \\
R_3 &\equiv F_2 (y_1, Q_1, y_2, Q_2, y_3, Q_3) &= 0 \\
R_4 &\equiv G_2 (y_1, Q_1, y_2, Q_2, y_3, Q_3) &= 0 \\
&- - - - - - - - - - \\
&- - - - - - - - - -
\end{aligned} \qquad (4.19)$$

$$R_{2i-1} \equiv F_i(y_{i-1}, Q_{i-1}, y_i, Q_i, y_{i+1}, Q_{i+1}) = 0$$

$$R_{2i} \equiv G_i(y_{i-1}, Q_{i-1}, y_i, Q_i, y_{i+1}, Q_{i+1}) = 0$$

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$$R_{2N-3} \equiv F_{N-1}(y_{N-2}, Q_{N-2}, y_{N-1}, Q_{N-1}, y_N, Q_N) = 0$$

$$R_{2N-2} \equiv G_{N-1}(y_{N-2}, Q_{N-2}, y_{N-1}, Q_{N-1}, y_N, Q_N) = 0$$

$$R_{2N-1} \equiv F_N(y_{N-1}, Q_{N-1}, y_N, Q_N) = 0$$

$$R_{2N} \equiv G_N(y_N, Q_N) = 0 \tag{4.19}$$

Although Eq. 4.19 carries 2N unknowns, yet each equation contains a maximum of six unknowns and the whole matrix can be put in a banded matrix of width seven. The generalized Newton iteration method is used herein to find the solution of this system. Needed partial derivatives and the solution may be obtained in the same way as described in finite difference method. (See Article 3.2). Knowing the values of dy_i and dQ_i, i having the value from 1 to N, the values of y_i and Q_i may be modified. The method used herein is

Gaussian Elimination method using banded matrix technique. At the terminal iteration, when values of dy_i and/or dQ_i are less than the permissible errors we take the values of variables found as final and advance for time stop (j+2). For this first we approximate the values of y_i and Q_i equal to (j+1) time level and then find the values of time derivatives. In the iterative cycles also the values of time derivatives are modified after the completion of each iteration.

4.3 PROGRAMMING

The above FEM method of numerical solution has been programed in FORTRAN-10 and implemented at DEC-1090 system at Indian Institute of Technology, Kanpur. Input and output procedure is the same as in Article 3.3. Provision for a large number of lateral flows has been made. The program can be used for a large number of distance and time nodes. Other programming details are the same as in Article 3.3. Flow chart of the program is shown in Fig. 3.2. A listing of the FORTRAN Program is included in APPENDIX - B.

4.4 CHARACTERISTICS OF FEM PROGRAM

In the present study the selection of distance steps has been done by the consideration of stability.

Δt and α should be selected such that the program is stable, convergent and gives accurate results. In the present study it has been found that $\alpha = 1.75$ satisfies the above properties to a large extent. The value of α should be selected between 1.5 - 2.0 such that it gives desired hydrograph.

In the present study verification has been done by the examples worked out by others and comparison of results is done among themselves and with them. If two or more sets of parameters give identical results, one consuming less CPU time is considered the best one. It has been observed that the results generally differ at the peak. The resultant hydrograph reproducing peak values, peak time and shape has been concluded the best. In this study the solution is taken as convergent, if it converges in 40 iteration steps.

CHAPTER - V

COMPUTER STUDIES

5.1 PERFORMANCE

Flood routing methods are used to produce the relevant outflows by computations. So the accuracy of the reproduction may be well defined by a criteria for deciding its performance. However, in most of the cases complete reproduction is not needed and only certain features will be of importance. In the present study as the real life floods data were not available an example worked out by Viesmann (17) has been adopted. So accuracy will be judged from that study only. The peak flow, the corresponding stage and the time to peak are the most important hydrograph characteristics. However the time to centroid of flow and flow volume may also constitute performance criterion. Eventhough effects of certain important routing parameters has been studied, but the effect of in between computation efforts has not been analysed. Further every care has been taken to minimize the computational efforts.

However the performance can not be rated only by the accuracy alone but by the convenience of use. Convenience can be judged by the computer time, simpleness

of the model and the range of applicability in the practical problems.

5.2 TESTING OF MODELS

The programs were first tested for programming errors. Some attention has been paid to find the limits of applicability. The numerical experimentation is done basically for different routing parametrs viz. $\frac{\Delta t}{\Delta x}$ and θ for finite difference method and $\frac{\Delta t}{\Delta x}$ and α for finite element method. The regular channel geometries were used. The following two examples were considered and tested with both the methods by using different downstream rating conditions.

5.3 EXAMPLE - I

The first example taken is the one as tested by Cooley and Moin(6) on their finite element model. It pertains to routing a traiangular discharge hydrograph down a rectangular channel and has been adopted from Viessman, et al.,(17). A rectangular channel 6.1 m wide and 3.2 km long carrying a steady uniform discharge of 23.34 m^3/S at 1.83 m depth is subjected to an upstream flood wave with a peak of 57 m^3/S increasing linearly in a period of 20 min. This upstream flow decreases

linearly from its peak to 23.34 m^3/S in 40 min. Additional properties are $S_o = 0.0015$ and $n = 0.02$.

The explicit method employed by Viessman, et al.[17] utilized a 2 sec. time-step size and a 160 m distance step up. Cooley and Moin[6] used time step of 1 min., 10 min. and irregular time steps varying between 4 min. to 10 min. They kept the distance interval same. In the present study the distance interval taken was 800 m and time steps tested were 1 min., 2.5 min., 5 min., and 7.5 min. The problem was solved by both finite difference method and finite element method. Both the methods were tested for the Kinematic as well as dynamic downstream boundary conditions. The resultant hydrographs have been compared among themselves and to those of Kiessman, et al.[17] and Cooley and Moin[6].

In finite difference program the test runs were taken for the value of $\theta = 0.5, 0.55, 0.60, 0.65$ and 1.00. The program was converging with all the values of θ for the time step of 5.00 min. in both the kinematic and dynamic cases. However for the value of $\theta = 0.5 - 0.65$ the program was also convergent for time step of 7.5 min. in the kinematic case. The minimum CPU time taken (1.13 seds) was in the kinematic case

for a value of $\theta = 0.65$. However the results were not so good for time step of 7.5 minutes. It is also a general observation that $\theta = 0.55 - 0.60$ gives better results even though the minimum CPU time is obtained for $\theta = 0.60 - 0.65$. $\theta = 1.0$ gave much higher CPU times. Hence it is thought that $\theta = 0.60$ may be adopted for getting good results as well as low CPU time. Time step of 5.0 minutes was adopted for comparison purposes. The CPU time taken is 1.59 sec. for kinematic case and 1.44 sec. for dynamic case.

The input and output discharge hydrographs at 1.6 km. and 3.2 km. downstream for $\Delta t = 5$ min and $\theta = 0.6$ for kinematic and dynamic FDM methods are plotted in Fig. 5.1. They are compared to the Viessman[17] and Cooley and Moin[6] hydrographs. The results obtained were in between these two methods and kinematic and dynamic cases gave similar discharges in this case.

The input and output stage hydrographs at 3.2 km downstream for the above cases are shown in Fig. 5.2. Dynamic models give lower stages and their peak time were more when compared to kinematic models.

In the finite element program the test runs were taken for $\alpha = 1.0, 1.25, 1.50, 1.75$ and 2.0. The CPU

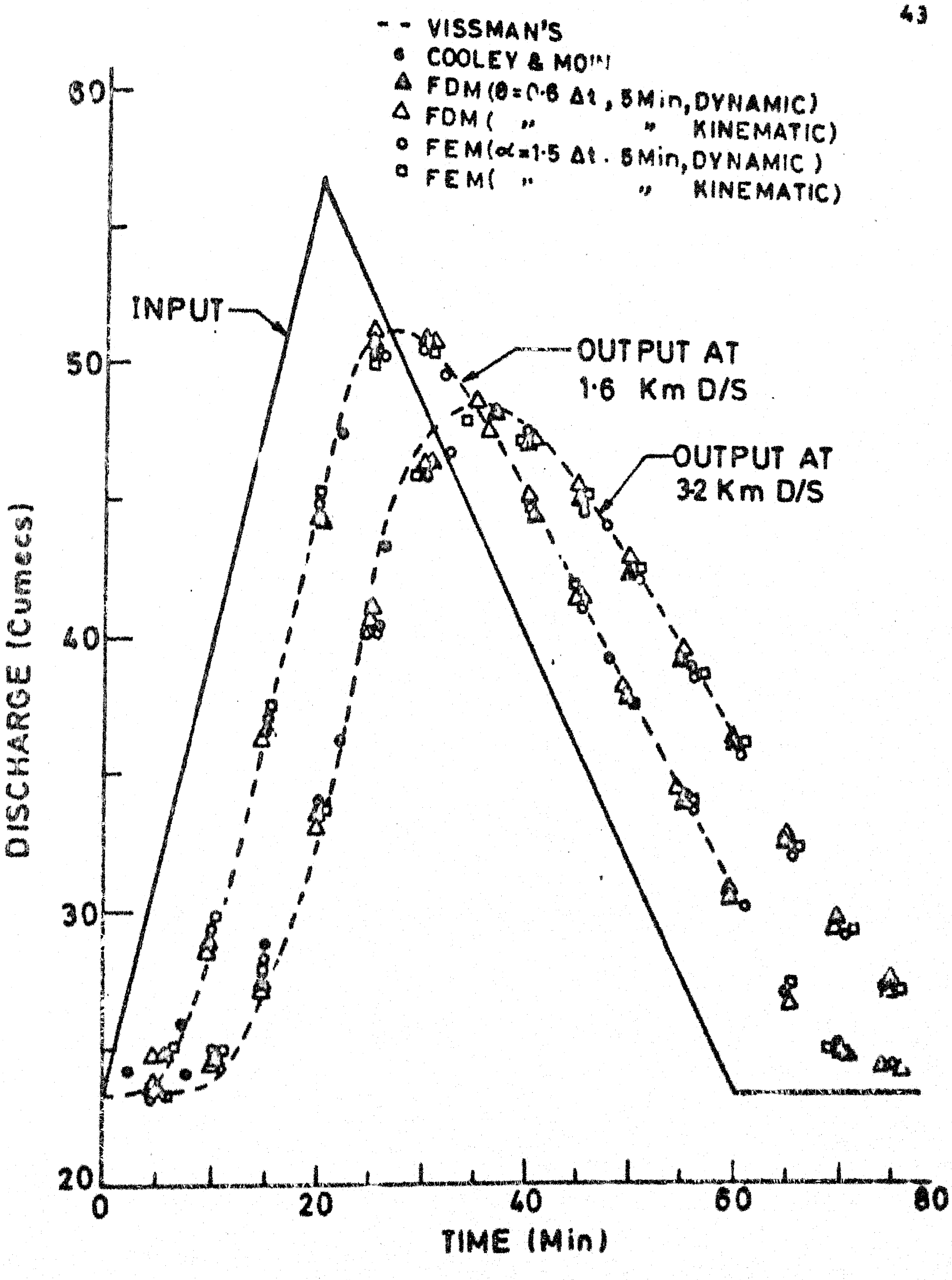

FIG.5.1 EXAMPLE I- OUTPUT DISCHARGE HYDROGRAPHS

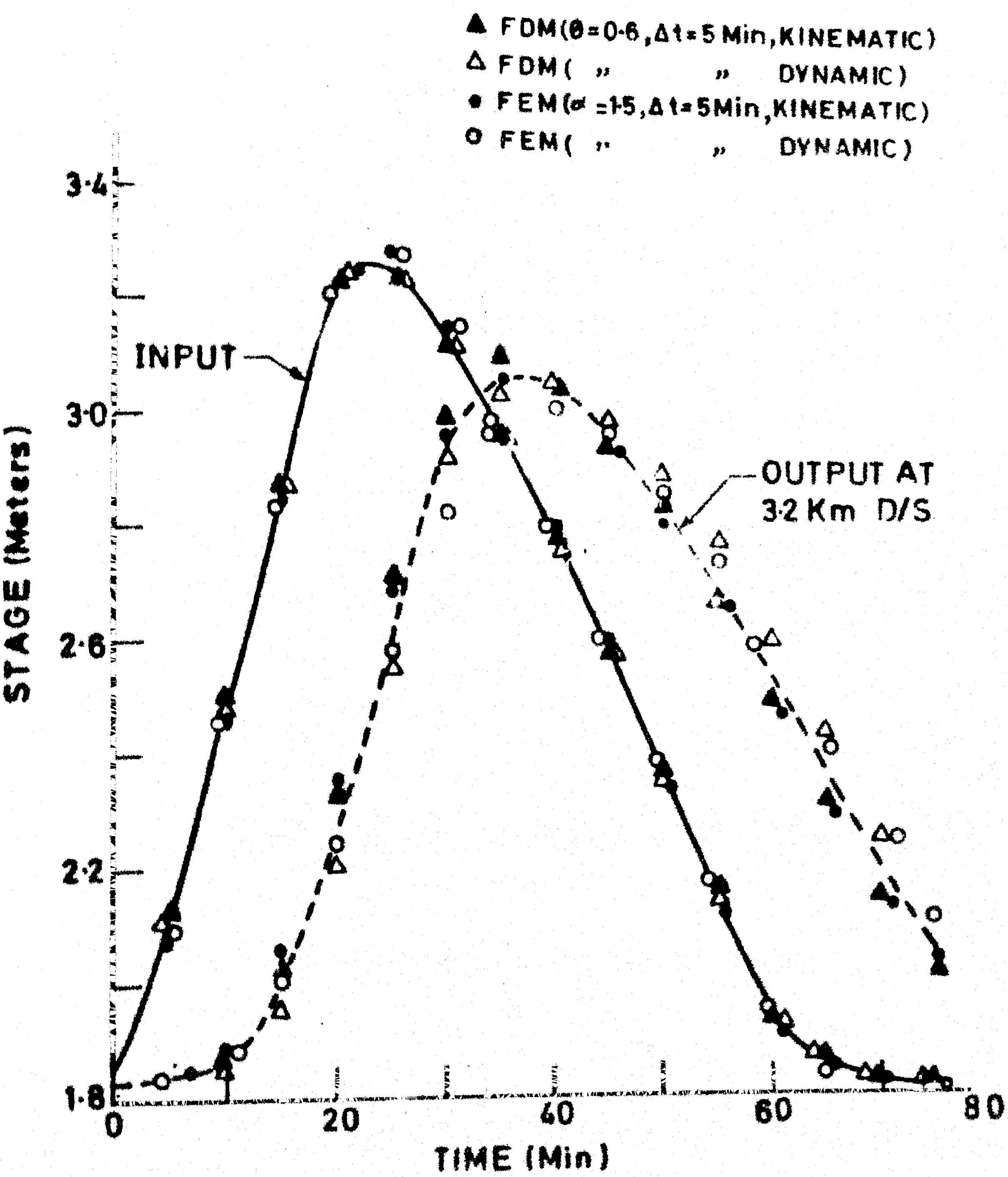

FIG.5·2 EXAMPLE I-OUTPUT STAGE HYDROGRAPHS

time for the converging output hydrograph is given in the Table 5.1. It is clear from the table that value of α does not change the reexcution efforts in the kinematic

TABLE 5.1 C.P.U. TIME FOR FEM PROGRAM (EXAMPLE I)

α	Kinematic			Dynamic	
	Δt = 1.0 min.	Δt = 2.5 min.	Δt = 5.0 min.	Δt = 2.5 min	Δt=5.0 min.
1.00	2.81	1.47	1.05	2.36	1.09
1.25	2.79	1.46	1.05	2.94	1.19
1.50	2.74	1.43	1.04	4.10	1.31
1.75	2.70	1.43	1.05	7.20	1.41
2.00	2.67	1.41	1.05	-	1.56

case. However in the dynamic case execution efforts are more for a higher value of α. CPU time is more in the dynamic case. The minimum CPU taken was for Δt = 5.0 min. and α = 1.50 in the kinematic case. The higher values of $\frac{\Delta t}{\Delta x}|V \pm c|$ are 0.56, 1.36 and 2.71 for the time steps of 1, 2.5 and 5.0 min. respectively and program was

stable even for $\frac{\Delta t}{\Delta x} |V \pm c|$ values greater than 1.0. The output hydrograph peak shifts to the right with the increasing values of α. The CPU time in the FEM model is less than that of FDM model.

Output discharge hydrographs at 1.6 km and 3.2 kms. downstream for $\Delta t = 5$ min. and $\alpha = 1.5$ are shown in Fig. 5.1 and are compared to other methods. FEM gives comparable results to other method. Kinematic and dynamic models gives similar results for this problem.

In Fig. 52 stage hydrographs for 1.6 km. downstream are given. FEM gave lower stage peak than FDM. Dynamic case gives lower peak and it occurs with a lag. FEM Kinematic model gives good results.

It is found that if time step is made near zero peaks of the discharge hydrographs at 1.6 km. downstream computed by FEM for different values of α were found to converge at a peak value of 51.4 cumecs. This value is approximately equal to Viessman's(17) results for 2 sec. time step explicit solution and hence it may be taken as time peak. Fig. 5.3 gives percentage results for higher values of time steps, if they are giving stable and convergent results. Dynamic and

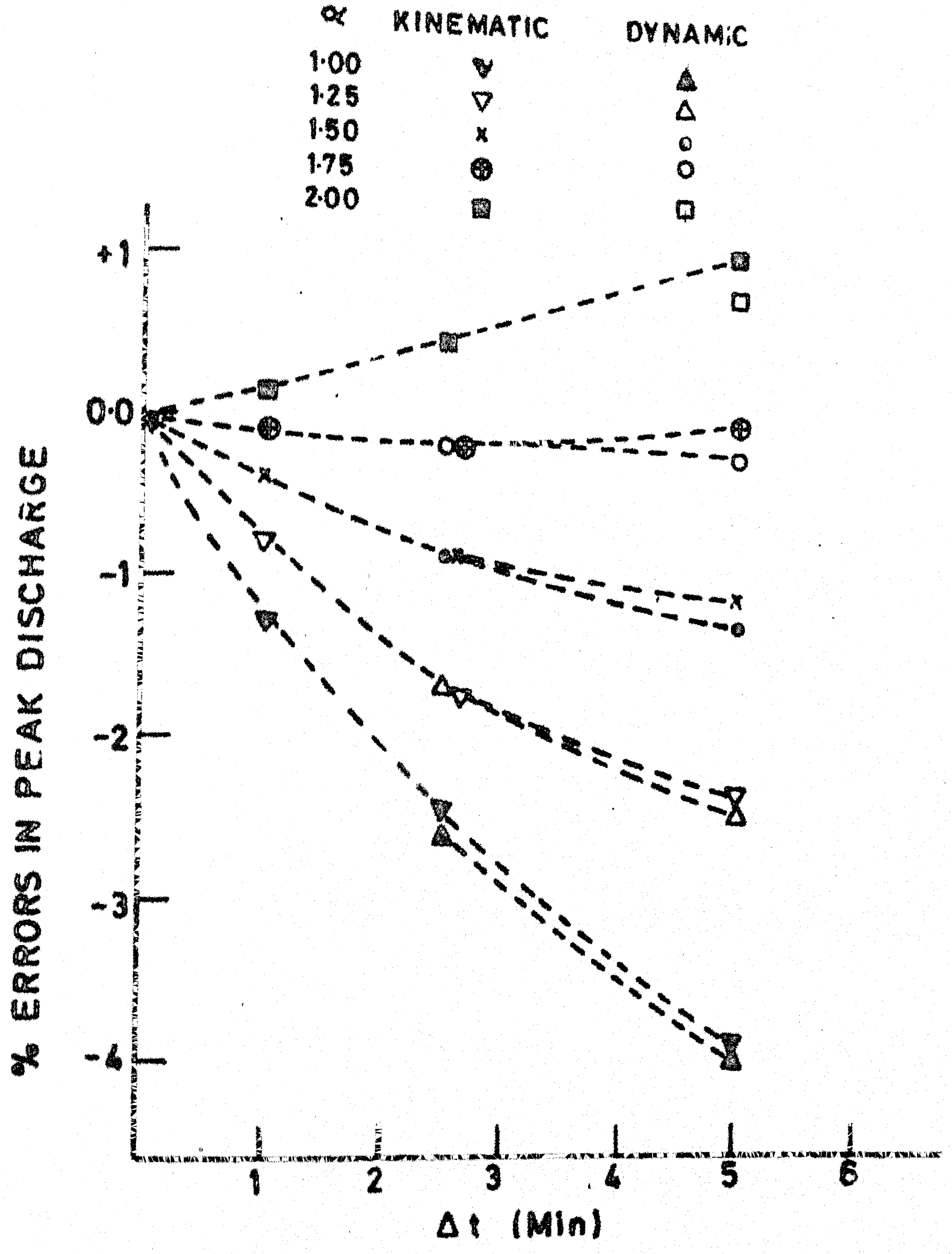

FIG.5·3 EXAMPLE I-ERRORS IN PEAK DISCHARGES OF HYDROGRAPHS AT 1·6 KM D/S IN FEM METHOD

Kinematic case are giving near about same error. α = 1 - 1.5 gave negative errors; α = 1.75 gave very less error; α = 2.0 gave positive errors. A value of α = 1.50 - 2.0 may be adopted. In this example α = 1.5 has been taken for comparison purposes while α = 2.0 has been taken in next example. Dynamic and kinematic cases gave similar errors.

5.4 EXAMPLE - II

In this example hypothetical input hydrograph described by the Peason Type III distribution has been used. The hydrograph is given by the equation

$$Q(t) = Q_b + (Q_p - Q_b)\left(\frac{t}{t_p}\right)^{\frac{1}{\beta-1}} e^{\frac{1}{\beta-1}\left(1-\frac{t}{t_p}\right)}$$

where Q_b = base flow = 350 m^3/sec.

Q_p = peak flow = 700 m^3/sec.

t_p = time of peak = 300 sec.

β = skewness factor = 1.15
(t_c/t_p)

and t_c = time of centroid of flow

The cross-sectional area and wetted parameter are given by the equations

$$A = 0.0008\, y^4 + 2y^3 - 10\, y^2 + 100\, y + 100$$

$$P = 0.0016\, y^3 + 4y^2 - 15\, y + 160$$

The Manning's n is 0.025 and bed slope is 0.0005. The channel is 20 km. long.

In the present study the distance interval taken was 2 kms. Problem was solved by both FDM and FEM. Both the methods were tested for the kinematic as well as dynamic down stream boundary conditions. The resultant stage and discharge were compared among themselves.

In the FDM program the test runs were taken for the value of $\theta = 0.5, 0.55, 0.60, 0.65$ and 1.0. The program was tested for $\Delta t = 21, 25, 40$ and 50 mins. Smaller values of time could not be tested due to limited matrix storage. The minimum computer CPU time was 2.51 secs. for $\Delta t = 50$ min. and $\theta = 0.65$. The kinematic case was not converging for $\theta = 1.0$ for $\Delta t > 25$ min. The dynamic case converged only at $\Delta t = 21$ minutes. The CPU time taken is 5.44 secs. for $\theta = 0.6$, $\Delta t = 21$ minutes, dynamic case and 3.17 secs. for $\theta = 0.6$, $\Delta t = 25$ min. kinematic case. These resultant stage and discharge hydrographs are plotted in Figs.5.4 and 5.5.

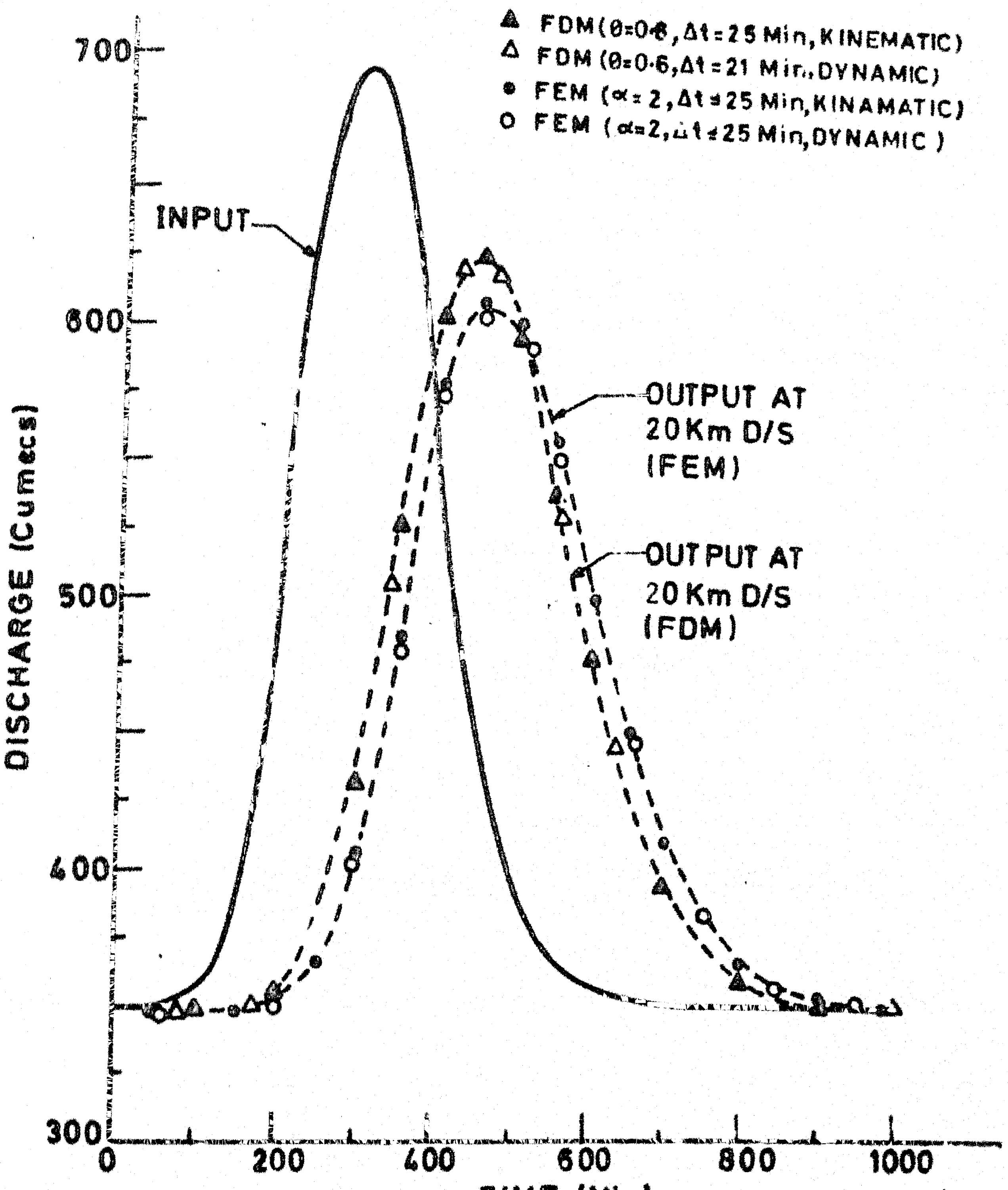

FIG.5·4 EXAMPLE II-OUTPUT DISCHARGE HYDROGRAPHS

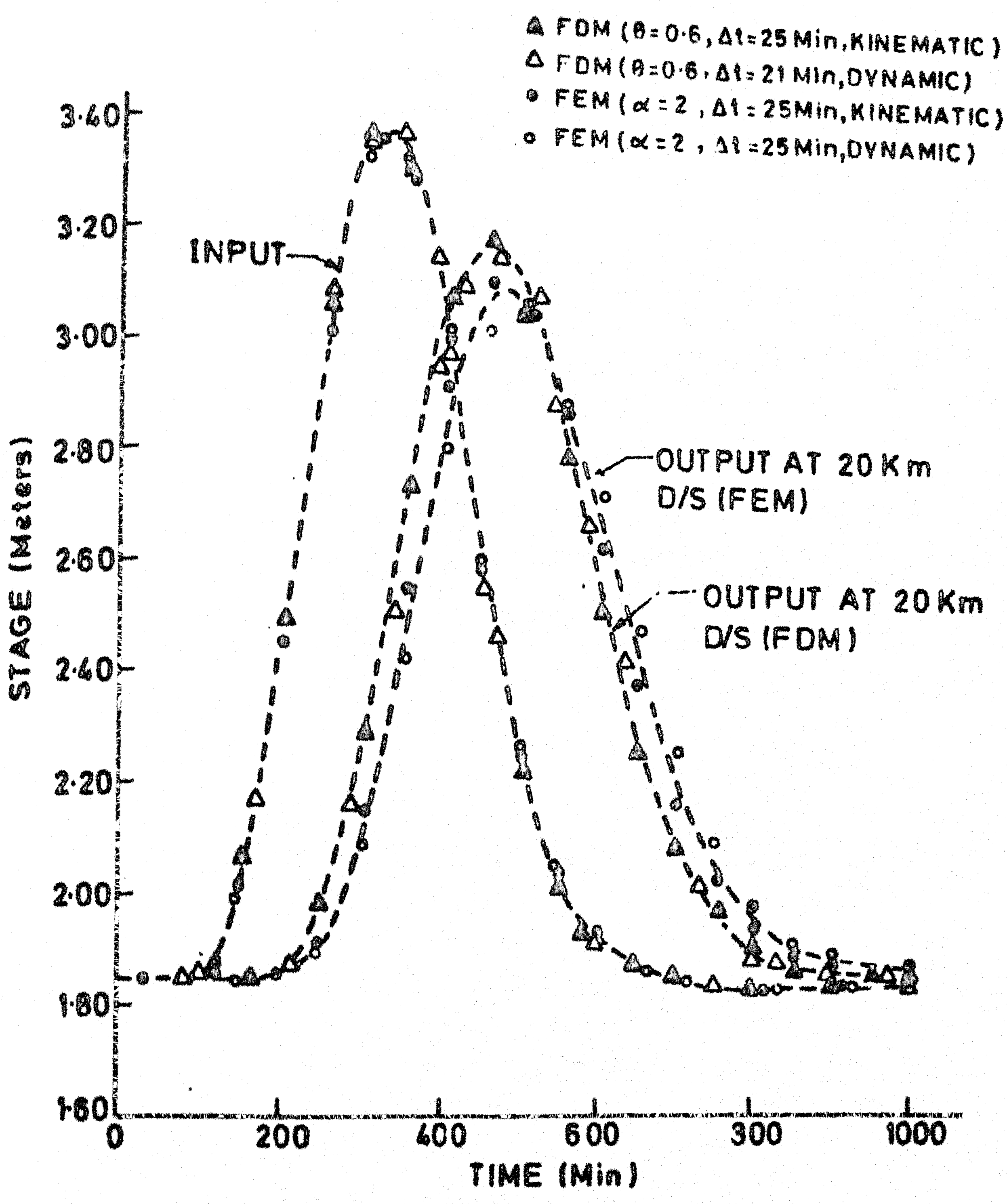

FIG.5.5 EXAMPLE II-OUTPUT STAGE HYDROGRAPHS

In the FEM Program the test runs were taken for α = 1.0, 1.25, 1.50, 1.75 and 2.0. The CPU time for converging and stable outputs is given in the Table 5.2. It can be seen from the table that CPU

TABLE 5.2 : CPU TIME FOR FEM PROGRAM (EXAMPLE II)

α	Kinematic				Dynamic			
	Δt =2.5 min.	Δt =5.0 min.	Δt =10.0 min.	Δt = 25.0 min.	Δt =2.5 min.	Δt =5.0 min.	Δt =10.0 min.	Δt =25.0 min.
1.00	25.02	14.85	11.07	-	26.55	15.31	11.64	-
1.25	24.77	14.59	10.08	-	-	15.19	10.53	-
1.50	24.42	14.46	9.52	11.18	-	14.90	9.86	11.67
1.75	23.69	14.24	9.26	8.80	-	16.70	9.59	8.88
2.00	23.25	13.85	9.10	7.69	-	-	9.49	7.22

time is more in the dynamic case. The minimum CPU taken was for Δt = 25 min. and α = 2.00.

Output stage and discharge hydrographs by FDM and FEM at 20 km. downstream for Δt = 25 min. and

and $\alpha = 2.0$ are shown in Fig. 5.4 and 5.5 and are compared among themselves, FEM is giving higher stage and discharge peaks than FEM. Kinematic case gives higher stage and discharge peaks. In dynamic case stage peak is time lagging.

It is seen that the program does not converges for smaller values of time steps. So proper care should be taken in the selection of Δt and $\alpha = 1.5 - 2.0$ may be used for computations.

CHPATER - VI

CONCLUSION

6.1 CONCLUSION

An FEM algorithm which blends the strong points of the two earlier studies has been developed in this study. This algorithm has been compared with the modified Amein's FDM Scheme. Both the methods were tested through numerical experimentation. The following significance conclusions are made.

(i) The proposed FDM which is based on Amein's 1970 work is stable, convergent for $\theta = 0.55 - 0.65$. $\theta = 0.6$ gives the best results.

(ii) The proposed FEM is found to give stable, convergent and accurate results for high time and distance steps. FEM program takes less CPU time while compared to FDM. It generally have advantage over FDM.

(iii) The value of $\alpha = 1.5 - 2.0$ gives stable, convergent and accurate results. A value of $\alpha = 1.75$ is recommended.

(iv) The stage hydrographs are more sensitive to the downstream boundary conditions while comparing with

discharge hydrographs. The peak stage in dynamic case is found time lagging.

6.2 FUTURE WORK

The proposed FEM has very good advantages like use of larger time and distance steps, less computational efforts while comparing to other methods. The following studies are needed.

(i) Further studies on the application to field problems.

(ii) Parametric study regarding selection of α for various hydrograph shapes and channel characteristics.

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APPENDEX-A

FDM PROGRAMME

MAIN PROGRAMME 'FDM' :

```
C        This programme rout a flood using Finite Difference method.
C     Upstream boundary condition may be either a stage hydrograph or
C     a flow hydrograph. Downstream boundary condition may be either
C     kinematic or dynamic. Channel properties and inflow hydrographs
C     are to be called from predefined subroutines. Global matrix is
C     stored in banded form and LEQT1B, a library subroutine of IMSL
C     is used for their solution. This programme is developed at DEC
C     -1091 system at I.I.T. Kanpur. Programming language is FORTRAN-
C     10.

C     Input Data :
Card  <UNIT>
Card  <S0,DX,DT,N,L,THETA,BC1,BC2,NS,DL>
Card  <Y(1,1)...Y(I,1)...Y(N,1)>
Card  <Q>
Card  <STN(1)...STN(I)...STN(N)>
Card  <EPSY,EPSV>
Card  < QINT > if  BC1 is 1.

      Where UNIT = 1 for MKS system of units
                 = 2 for FPS system of units
            N    =Number of nodes on X axis
            L    =Number of nodes on t axis
            BC1  = 1 if upstream stage hydrograph is given
                 = 2 if upstream discharge hydrograph is given
            BC2  = 1 if rating curve is kinematic
                 = 2 if rating curve is dynamic
            NS   =Number of nodes where output is needed
            DL   =Interval of time steps after which output is needed
            Y(I,J)=Flow depth at Ith node & Jth time level
            DT   =Time increament in minutes
           STN(I)=Ith node number at which output is needed
           QINT  =Initial discharge
           EPSY  =Permissible error in flow depth
           EPSV  =Permissible error in velocity
           S0    =Bed slope
           THETA =Weighting factor(0.5-1.0)
           Q     =Lateral discharge per unit length

      REAL INHYD,MAN,MAT
      INTEGER COND,BC1,BC2,UNIT,STN,DL
      DIMENSION AR(40,50),BR(40,50),PR(40,50),MAN(40,50)
     1,SF(40,50),MAT(80,5),R(80),V(40,50),Y(40,50),
     2X(40),T(50),INHYD(50),XL(240),STN(40)
      DIMENSION A(40),B(40),C(40),D(40),E(40),H(40),O(40),
     1P(40),W(40),A1(40),B1(40),C1(40),D1(40),E1(40),
     2H1(40),O1(40),QOUT(40,50)
      COMMON X,Y
      F(I)=AI+A(I)+DTX*((CI+C(I))*
     1(BI+B(I))+(HI+H(I))*(EI+E(I)))/4.+
     3DT/2.*Q*(WI+W(I))
      G(I)=(AI1+A1(I))+DXT/GR*(BI1+B1(I)
     1)+0.25/GR*(CI1+C1(I))*(DI1+D1(I))
     2+0.5*DX*(HI1+H1(I))+0.5*DX*Q/GR*
     3(OI1+O1(I))
      DFY(I)=1-DTX*0.25*((BI+B(I))+((
     1EI+E(I))*THETA*(1.-AR(I,J+1)/BR(I,J+1)**2*DBY(I,J+1))))-
     2Q/2.*DT/BR(I,J+1)**2*DBY(I,J+1)*THETA
      DFY1(I)=1+DTX*0.25*((BI+B(I))+((
     1EI+E(I))*THETA*(1.-AR(I+1,J+1)/BR(I+1,J+1)**2*DBY(I+1,J+1))))-
     2Q/2.*DT/BR(I+1,J+1)**2*DBY(I+1,J+1)*THETA
      DFV(I)=0.25*DTX*((CI+C(I))-(
     1HI+H(I)))
      DFV1(I)=0.25*DTX*((CI+C(I))+(
     1HI+H(I)))
```

```
      DGY1(I)=(1+2./3.*DX*SF(I+1,J+1)*(1./PR(I+1,J+1)*DPY(T+1,J+1)
     1-RR(I+1,J+1)/AR(I+1,J+1))-Q*DX*0.5/GR*V(I+1,J+1)*RR(I+1,J+1)/
     2AR(I+1,J+1)**2)*THETA
      DGV(I)=DXT/GR+0.25/GR*((D1(I)-C1(I))*THETA-2*V(I,J+1)*THETA
     1**2)+DX*SF(I,J+1)/V(I,J+1)*THETA+DX*0.5/GR*Q/AR(I,J+1)*V(I,J+1
     2)*THETA
      DGV1(I)=DXT/GR+0.25/GR*((C1(I)+D1(I))*THETA+2.*V(I+1,J+1)
     1*THETA**2)+DX*THETA*SF(I+1,J+1)/V(I+1,J+1)+
     2DX*0.5/GR*Q/AR(I+1,J+1)*V(I+1,J+1)*THETA
501   FORMAT(5X,'Numbers of nodes on X axis =',I5,/,5X,'Number o
     1f nodes on t axis =',I5,/,5X,'Time step =',F6.2,' Minute')
402   FORMAT(5X,'Bottom Slope =',F12.9)
502   FORMAT(/,5X,'Distance(Met.)',
     15X,'Initial Stage(Met.)'/,(7X,F10.2,15X,F10.2))
602   FORMAT(/,5X,'Distance(Feet)',
     15X,'Initial Stage(Feet)'/,(7X,F10.2,15X,F10.2))
503   FORMAT(/,5X,'Lateral Flow =',F12.2,' Cumecs/Meter')
603   FORMAT(/,5X,'Lateral Flow =',F12.2,' Cusecs/Feet')
504   FORMAT(/,5X,'EPSY =',F12.6,/,5X,'EPSV =',F12.6)
505   FORMAT(10X,'FLOOD ROUTING BY FINITE DIFF. METHOD - KINE
     1MATIC MODEL',/,10X,56('_'))
506   FORMAT(10X,'FLOOD ROUTING BY FINITE DIFF. METHOD - DYNA
     1MIC MODEL',/,10X,54('_'))
507   FORMAT(/,5X,'Stage hydrograph is given at upstream')
508   FORMAT(/,5X,'Flow hydrograph is given at upstream')
512   FORMAT(/,5X,'Station at',F10.2,'  Meters')
612   FORMAT(/,5X,'Station at',F10.2,'  Feet')
513   FORMAT(5X,'S.N.',5X,'Time(Minute)',5X,'Stage(Met.)',5X
     1,'Discharge(Cumecs)')
613   FORMAT(5X,'S.N.',5X,'Time(Minute)',5X,'Stage(Feet)',5X
     1,'Discharge(Cusecs)')
514   FORMAT(5X,I4,5X,F8.2,10X,F6.2,10X,F12.2)
516   FORMAT(5X,'HYDROGRAPHS :')
517   FORMAT(5X,'NOT CONVERGING')
518   FORMAT(5X,'THETA =',F5.2)
519   FORMAT(5X,'Average velocity =',F8.3,' Met/Sec',/,
     15X,'Average Celerity =',F8.3,'Met/Sec')
619   FORMAT(5X,'Average velocity =',F8.3,' Feet/Sec',/,
     15X,'Average Celerity =',F8.3,'Feet/Sec')
520   FORMAT(5X,'Maximum number of iteration used =',I3)
      READ*,UNIT
      GR=9.81
      IF(UNIT.EQ.2)GR=32
      READ*,SO,DX,DT,N,L,THETA,BC1,BC2,NS,DL
      READ*,(Y(I,1),I=1,N)
      READ*,Q
      READ*,(STN(I),I=1,NS)
      READ*,EPSY,EPSV
      IF(BC2.EQ.1)TYPE 505
      IF(BC2.EQ.2)TYPE 506
      IF(BC1.EQ.1)TYPE 507
      IF(BC1.EQ.2)TYPE 508
      TYPE 501,N,L,DT
      TYPE 402,SO
      NN=N*2
      J=1
      K=0
      MIT=K
      X(1)=0;T(1)=0
      INHYD(1)=HYDR(T(1))
      IF(BC1.EQ.1)GO TO 3
      QINT=INHYD(1)
      GO TO 4
3     READ*,QINT
4     V(1,J)=QINT/AREA(1,J)
      QOUT(1,1)=QINT
      CEL=SQRT(GR*Y(1,J))
      AVV=V(1,J)
```

```
      V(I,J)=QINT/AREA(LOCAL,J)
      QOUT(I,J)=QINT
      CEL=CEL+SQRT(GR*Y(I,J))
      AVV=AVV+V(I,J)
1     X(I)=X(I-1)+DX
      IF(UNIT.EQ.1)TYPE 502,(X(I),Y(I,1),I=1,N)
      IF(UNIT.EQ.2)TYPE 602,(X(I),Y(I,1),I=1,N)
      IF(UNIT.EQ.1)TYPE 503,Q
      IF(UNIT.EQ.2)TYPE 603,Q
      TYPE 504,EPSY,EPSV
      TYPE 518,THETA
      THETA=THETA*2.
      RTHETA=2.-THETA
      DO 2 I=2,L
      T(I)=T(I-1)+DT
      LOCAL=I
      INHYD(I)=HYDR(T(LOCAL))
2     CONTINUE
      DT=DT*60
      DTX=DT/DX;DXT=DX/DT
      TYPE 516
11    DO 12 I=1,N
      LOCAL=I
      AR(I,J)=AREA(LOCAL,J)
      BR(I,J)=BREATH(LOCAL,J)
      PR(I,J)=PARAM(LOCAL,J)
      MAN(I,J)=FRICT(LOCAL,J)
12    SF(I,J)=MAN(I,J)**2*V(I,J)*ABS(V(I,J))*(PR(I,J)/
     1AR(I,J))**(4./3.)
21    DO 20 I=1,N-1
      A(I)=(-Y(I+1,J)-Y(I,J))
      B(I)=RTHETA*(V(I,J)+V(I+1,J))
      C(I)=RTHETA*(Y(I+1,J)-Y(I,J))
      E(I)=RTHETA*(V(I+1,J)-V(I,J))
      H(I)=RTHETA*(AR(I,J)/BR(I,J)+AR(I+1,J)/BR(I+1,J))
      W(I)=RTHETA*(1./BR(I,J)+1./BR(I+1,J))
      A1(I)=C(I)
      B1(I)=-V(I,J)-V(I+1,J)
      C1(I)=B(I)
      D1(I)=E(I)
      H1(I)=RTHETA*(SF(I,J)+SF(I+1,J))-4*S0
      D1(I)=RTHETA*(V(I,J)/AR(I,J)+V(I+1,J)/AR(I+1,J))
      V(I,J+1)=V(I,J)
20    Y(I,J+1)=Y(I,J)
      IF(BC1.EQ.2)GO TO 26
      Y(1,J+1)=INHYD(J+1)
      GO TO 27
26    V(1,J+1)=INHYD(J+1)/AREA(1,J+1)
27    Y(N,J+1)=Y(N,J)
      V(N,J+1)=V(N,J)
31    DO 32 I=1,N
      LOCAL=I
      AR(I,J+1)=AREA(LOCAL,J+1)
      BR(I,J+1)=BREATH(LOCAL,J+1)
      PR(I,J+1)=PARAM(LOCAL,J+1)
      MAN(I,J+1)=FRICT(LOCAL,J+1)
      SF(I,J+1)=MAN(I,J+1)**2*V(I,J+1)*ABS(V(I,J+1))*(PR(I,J+1)/
     1AR(I,J+1))**(4./3.)
32    CONTINUE
      QO=AR(N,J+1)*V(N,J+1)
      QN=1/MAN(N,J+1)*AR(N,J+1)**(5./3.)/PR(N,J+1)**(2./3.)*S0**0.5
      MAT(1,1)=0;MAT(1,2)=0
      IF(BC1.EQ.2)GO TO 35
      MAT(1,3)=1
      MAT(1,4)=0
      GO TO 36
35    MAT(1,3)=-V(1,J+1)*BR(1,J+1)
      MAT(1,4)=-AR(1,J+1)
```

```
      R(1)=0
      IF(BC1.EQ.2)R(1)=-TNHYD(J+1)+AR(1,J+1)*V(1,J+1)
      MAT(NN,1)=0
      IF(BC2.EQ.2)GO TO 37
      MAT(NN,2)=V(N,J+1)*BR(N,J+1)+1/MAN(N,J+1)*(AR(N,J+1)**(5./3.)*DP
     1Y(N,J+1)*2/3/PR(N,J+1)**(5./3.)-5/3*AR(N,J+1)**(2./
     23.)*BR(N,J+1)/PR(N,J+1)**(2./3.))*SQRT(S0)
      MAT(NN,3)=AR(N,J+1)
      R(NN)=-Q0+QN
      GO TO 38
37    QFAC=SQRT(1-1/S0*(Y(N,J+1)-Y(N-1,J+1))/DX-V(N,J+1)/S0/GR
     1*(V(N,J+1)-V(N-1,J+1))/DX-1/S0/GR*(V(N,J+1)-V(N,J))/DT)
      MAT(NN,2)=V(N,J+1)*BR(N,J+1)+(1/MAN(N,J+1)*(AR(N,J+1)**(5./3.)*D
     1PY(N,J+1)*2/3/PR(N,J+1)**(5./3.)-5/3*AR(N,J+1)**(2./
     23.)*BR(N,J+1)/PR(N,J+1)**(2./3.))*SQRT(S0))*QFAC
     3+QN*0.5/QFAC/S0/DX
      MAT(NN,3)=AR(N,J+1)-0.5*QN/QFAC*(-1/S0/DX/GR*(2*V(N,J+1)-
     1V(N-1,J+1))-1/S0/GR/DT)
      R(NN)=-Q0+QN*QFAC
38    DO 41 I=1,N-1
      II=I*2;LOCAL=1
      AI=(Y(I+1,J+1)+Y(I,J+1))
      BI=THETA*(V(I,J+1)+V(I+1,J+1))
      CI=THETA*(Y(I+1,J+1)-Y(I,J+1))
      EI=THETA*(V(I+1,J+1)-V(I,J+1))
      HI=THETA*(AR(I,J+1)/BR(I,J+1)+AR(I+1,J+1)/BR(I+1,J+1))
      WI=THETA*(1./BR(I,J+1)+1./BR(I+1,J+1))
      AI1=CI
      BI1=V(I,J+1)+V(I+1,J+1)
      CI1=BI
      DI1=EI
      HI1=THETA*(SF(I,J+1)+SF(I+1,J+1))
      DI1=THETA*(V(I,J+1)/AR(I,J+1)+V(I+1,J+1)/AR(I+1,J+1))
      R(II)=-F(LOCAL)
      R(II+1)=-G(LOCAL)
      MAT(II,1)=0
      MAT(II,2)=DFY(LOCAL)
      MAT(II,3)=DFV(LOCAL)
      MAT(II,4)=DFY1(LOCAL)
      MAT(II,5)=DFV1(LOCAL)
      II=II+1
      MAT(II,1)=DGY(LOCAL)
      MAT(II,2)=DGV(LOCAL)
      MAT(II,3)=DGY1(LOCAL)
      MAT(II,4)=DGV1(LOCAL)
      MAT(II,5)=0
41    CONTINUE
      CALL LEQT1B(MAT,NN,2,2,80,R,1,80,0,XL,IER)
      COND=0
      DO 51 I=1,N
      LOCAL=2*I
      Y(I,J+1)=Y(I,J+1)+R(LOCAL-1)
      V(I,J+1)=V(I,J+1)+R(LOCAL)
      IF(ABS(R(LOCAL-1)).GT.EPSY.OR.ABS(R(LOCAL)).GT.EPSV)
     1COND=1
51    CONTINUE
      K=K+1
      IF(MIT.LT.K)MIT=K
      IF(K.EQ.40)GO TO 95
      IF(COND.EQ.1)GOTO31
      K=0
      DO 60 I=1,N
      CEL=CEL+SQRT(GR*Y(I,J+1))
      AVV=AVV+V(I,J+1)
60    QOUT(I,J+1)=V(I,J+1)*AR(I,J+1)
61    J=J+1
      IF(J.NE.L)GO TO 11
      DO 91 I=1,NS
```

```
      IF(UNIT.EQ.1)TYPE 513
      IF(UNIT.EQ.2)TYPE 613
91    TYPE 514,((J,T(J),Y(STN(I),J),OUHT(STN(I),J)),J=1,L,DL)
      AVV=AVV/(N*L)
      CEL=CEL/(N*L)
      IF(UNIT.EQ.1)TYPE 519,AVV,CEL
      IF(UNIT.EQ.2)TYPE 519,AVV,CEL
      TYPE 520,NIT
      STOP
95    TYPE 517
      STOP
      END
```

```
              EXAMPLE-I
        FUNCTIONS/SUBROUTINE '10'

        FUNCTION AREA(I,J)
        COMMON X,Y
        DIMENSION X(40),Y(40,50)
        AREA=6.1*Y(I,J)
        RETURN
        END
        FUNCTION BREATH(I,J)
        COMMON X,Y
        DIMENSION X(40),Y(40,50)
        BREATH=6.1
        RETURN
        END
        FUNCTION DBY(I,J)
        COMMON X,Y
        DIMENSION X(40),Y(40,50)
        DBY=0
        RETURN
        END
        FUNCTION PARAM(I,J)
        COMMON X,Y
        DIMENSION X(40),Y(40,50)
        PARAM=6.1+2*Y(I,J)
        RETURN
        END
        FUNCTION DPY(I,J)
        COMMON X,Y
        DIMENSION X(40),Y(40,50)
        DPY=2
        RETURN
        END
        FUNCTION FRICT(I,J)
        COMMON X,Y
        DIMENSION X(40),Y(40,50)
        FRICT=0.02
        RETURN
        END
        SUBROUTINE LATFLW(NO,J,QTRIB,UX)
        DIMENSION QTRIB(30),UX(30)
        DO 1 I=1,NO
        QTRIB(I)=200
 1      UX(I)=0
        RETURN
        END
        FUNCTION HYDR(T)
        IF(T.GT.20)GO TO 1
        HYDR=23.34+1.683*T
        RETURN
 1      IF(T.GE.60)GO TO 2
        HYDR=23.34+(60-T)*0.8415
        RETURN
 2      HYDR=23.34
        RETURN
        END
```

INPUT DATA(KINEMATIC) '1D1.CDR' :

```
1
0.0015 800 5.5 16 0.6 2 1   3 1
1.83 1.83 1.83 1.83 1.83
0.0
1 3 5
0.001 1
```

OUTPUT '101.DAT' :

FLOOD ROUTING BY FINITE DIFF. METHOD - KINEMATIC MODEL

Flow hydrograph is given at upstream
Numbers of nodes on X axis = 5
Number of nodes on t axis = 16
Time step = 5.00 Minute
Bottom Slope = 0.001500000

Distance(Met.)	Initial Stage(Met.)
0.00	1.83
800.00	1.83
1600.00	1.83
2400.00	1.83
3200.00	1.83

Lateral Flow = 0.00 Cumecs/meter

EPSY = 0.001000
EPSV = 1.000000
THETA = 0.60
HYDROGRAPHS :

Station at 0.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.83	23.34
2	5.00	2.12	31.76
3	10.00	2.50	40.17
4	15.00	2.87	48.59
5	20.00	3.24	57.00
6	25.00	3.24	52.79
7	30.00	3.13	48.58
8	35.00	2.97	44.38
9	40.00	2.79	40.17
10	45.00	2.59	35.96
11	50.00	2.38	31.75
12	55.00	2.16	27.55
13	60.00	1.93	23.34
14	65.00	1.86	23.34
15	70.00	1.83	23.34
16	75.00	1.82	23.34

Station at 1600.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.83	23.34
2	5.00	1.86	24.50
3	10.00	2.03	28.79
4	15.00	2.37	36.89
5	20.00	2.72	44.84
6	25.00	3.02	51.21
7	30.00	3.11	51.07
8	35.00	3.07	48.13
9	40.00	2.98	45.30
10	45.00	2.84	41.66
11	50.00	2.68	38.08
12	55.00	2.50	34.24
13	60.00	2.30	30.39
14	65.00	2.11	26.92
15	70.00	1.98	24.96
16	75.00	1.91	24.36

Station at 3200.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.83	23.34
2	5.00	1.84	23.76
3	10.00	1.87	24.41
4	15.00	2.03	27.31
5	20.00	2.34	33.33
6	25.00	2.71	40.74
7	30.00	2.99	46.56
8	35.00	3.10	48.72
9	40.00	3.05	47.80
10	45.00	2.96	45.93
11	50.00	2.83	43.12
12	55.00	2.67	39.88
13	60.00	2.50	35.47
14	65.00	2.32	32.90
15	70.00	2.15	29.61
16	75.00	2.02	27.10

Average Velocity = 2.358 Met/Sec
Average Celerity = 4.884Met/Sec
Maximum number of iteration used = 9
STOP

```
          INPUT(DYNAMIC) '102.COR'

1
0.0015 800 5.5 16 0.6 2 2  3 1
1.83 1.83 1.83 1.83 1.83
0.0
1 3 5
0.001 1
```

OUTPUT :

FLOOD ROUTING BY FINITE DIFF. METHOD - DYNAMIC MODEL

Flow hydrograph is given at upstream
Numbers of nodes on X axis = 5
Number of nodes on t axis = 16
Time step = 5.00 Minute
Bottom Slope = 0.001500000

Distance(Met.)	Initial Stage(Met.)
0.00	1.83
800.00	1.83
1600.00	1.83
2400.00	1.83
3200.00	1.83

Lateral Flow = 0.00 Cumecs/Meter

EPSY = 0.001000
EPSV = 1.000000
THETA = 0.60
HYDROGRAPHS :

Station at 0.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.83	23.34
2	5.00	2.12	31.76
3	10.00	2.50	40.17
4	15.00	2.87	48.58
5	20.00	3.24	57.00
6	25.00	3.24	52.79
7	30.00	3.13	48.59
8	35.00	2.97	44.38
9	40.00	2.79	40.17
10	45.00	2.59	35.96
11	50.00	2.38	31.75
12	55.00	2.16	27.55
13	60.00	1.93	23.34
14	65.00	1.86	23.34
15	70.00	1.83	23.35
16	75.00	1.82	23.34

Station at 1600.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.83	23.34
2	5.00	1.86	24.50
3	10.00	2.03	28.82
4	15.00	2.36	36.90
5	20.00	2.72	44.81
6	25.00	3.02	51.15
7	30.00	3.12	51.05
8	35.00	3.07	48.17
9	40.00	2.97	45.40
10	45.00	2.84	41.65
11	50.00	2.68	38.08
12	55.00	2.49	34.22
13	60.00	2.30	30.36
14	65.00	2.11	26.92
15	70.00	1.98	24.95
16	75.00	1.91	24.36

Station at 3200.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.83	23.34
2	5.00	1.84	23.75
3	10.00	1.85	24.57
4	15.00	1.95	27.61
5	20.00	2.21	33.74
6	25.00	2.56	41.07
7	30.00	2.87	46.59
8	35.00	3.04	48.21
9	40.00	3.05	47.34
10	45.00	3.00	45.42
11	50.00	2.90	42.72
12	55.00	2.77	39.63
13	60.00	2.61	36.28
14	65.00	2.44	32.86
15	70.00	2.26	29.70
16	75.00	2.10	27.35

Average velocity = 2.357 Met/Sec
Average Celerity = 4.885Met/Sec
Maximum number of iteration used = 9
STOP

```
        EXAMPLE-II
SUBROUTINE/FUNCTIONS '2D' :

FUNCTION AREA(I,J)
DIMENSION X(40),Y(40,50)
COMMON X,Y
AREA=0.0008*Y(I,J)**4+2*Y(I,J)**3-10*Y(I,J)**2+100*Y(I,J)+100
RETURN
END
FUNCTION BREATH(I,J)
DIMENSION X(40),Y(40,50)
COMMON X,Y
BREATH=0.0032*Y(I,J)**3+6*Y(I,J)**2-10
1*Y(I,J)+100
RETURN
END
FUNCTION PARAM(I,J)
DIMENSION X(40),Y(40,50)
COMMON X,Y
PARAM=0.0016*Y(I,J)**3+4.0*Y(I,J)**2-15*Y(I,J)+
1160
RETURN
END
FUNCTION FRICT(I,J)
DIMENSION X(40),Y(40,50)
COMMON X,Y
FRICT=0.025
RETURN
END
FUNCTION DBY(I,J)
DIMENSION X(40),Y(40,50)
COMMON X,Y
DBY=0.0096*Y(I,J)**2+12*
1Y(I,J)-10
RETURN
END
FUNCTION DPY(I,J)
DIMENSION X(40),Y(40,50)
COMMON X,Y
DPY=0.004*Y(I,J)**2+6.4*
1Y(I,J)-15
RETURN
END
FUNCTION HYDR(T)
DATA QB/350/,QP/700/,TP/300/,GAMA/1.15/
HYDR=QB+(QP-QB)*(T/TP)**(1./(GAMA-1.)/EXP(GAMA-1.)*(1.-T/TP))
RETURN
END
SUBROUTINE LATFLW(NQ,J,QTRIB,UX)
DIMENSION QTRIB(30),UX(30)
RETURN
END
```

INPUT DATA(KINEMATIC) 'ZD1.CDR' :

```
1
0.0005 2000 25 11 41 0.6 2 1 3 2
1.85 1.85 1.85 1.85 1.85 1.85 1.85 1.85 1.85 1.85 1.85
0.0
1 6 11
0.005 1.0
```

OUTPUT '201.DAT' :

FLOOD ROUTING BY FINITE DIFF. METHOD - KINEMATIC MODEL

Flow hydrograph is given at upstream
Numbers of nodes on X axis = 11
Number of nodes on t axis = 41
Time step = 25.00 Minute
Bottom Slope = 0.000500000

Distance(Met.)	Initial Stage(Met.)
0.00	1.85
2000.00	1.85
4000.00	1.85
6000.00	1.85
8000.00	1.85
10000.00	1.85
12000.00	1.85
14000.00	1.85
16000.00	1.85
18000.00	1.85
20000.00	1.85

Lateral Flow = 0.00 Cumecs/Meter

EPSY = 0.005000
EPSV = 1.000000
THETA = 0.60
HYDROGRAPHS :

Station at 0.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.85	350.00
3	50.00	1.85	349.98
5	100.00	1.87	355.12
7	150.00	2.04	397.49
9	200.00	2.50	511.00
11	250.00	3.07	643.70
13	300.00	3.38	699.85
15	350.00	3.32	652.06
17	400.00	2.99	551.96
19	450.00	2.57	460.05
21	500.00	2.23	399.61
23	550.00	2.02	369.30
25	600.00	1.92	356.56
27	650.00	1.88	351.98
29	700.00	1.86	350.54
31	750.00	1.86	350.13
33	800.00	1.85	350.03
35	850.00	1.85	350.01
37	900.00	1.85	350.00
39	950.00	1.85	350.00
41	1000.00	1.85	350.00

Station at 10000.00 meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.85	350.00
3	50.00	1.85	349.54
5	100.00	1.85	350.06
7	150.00	1.87	355.01
9	200.00	1.99	385.30
11	250.00	2.34	469.79
13	300.00	2.84	584.34
15	350.00	3.19	654.31
17	400.00	3.25	645.24
19	450.00	3.06	581.70
21	500.00	2.75	505.01
23	550.00	2.44	441.02
25	600.00	2.19	397.57
27	650.00	2.02	372.36
29	700.00	1.93	359.59
31	750.00	1.89	353.83
33	800.00	1.87	351.46
35	850.00	1.86	350.54
37	900.00	1.86	350.19
39	950.00	1.85	350.12
41	1000.00	1.85	350.04

Station at 20000.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.85	350.00
3	50.00	1.85	349.11
5	100.00	1.85	349.39
7	150.00	1.85	349.91
9	200.00	1.87	353.85
11	250.00	1.98	374.32
13	300.00	2.29	433.91
15	350.00	2.74	528.17
17	400.00	3.09	606.52
19	450.00	3.19	628.02
21	500.00	3.06	598.74
23	550.00	2.80	540.93
25	600.00	2.52	479.99
27	650.00	2.27	429.70
29	700.00	2.09	394.36
31	750.00	1.98	372.60
33	800.00	1.91	361.15
35	850.00	1.88	354.93
37	900.00	1.87	351.64
39	950.00	1.86	351.37
41	1000.00	1.86	350.35

Average Velocity = 1.419 Met/Sec
Average Celerity = 4.649Met/Sec
Maximum number of iteration used = 4
STOP

```
INPUT(DYNAMIC) '202.CUR' :
1
0.0005 2000 21 11 48 0.6 2 2 3 2
1.85 1.85 1.85 1.85 1.85 1.85 1.85 1.85 1.85 1.85 1.85
0.0
1 6 11
0.005 1.0
```

OUTPUT '202.DAT' :

FLOOD ROUTING BY FINITE DIFF. METHOD - DYNAMIC MODEL

Flow hydrograph is given at upstream
Numbers of nodes on X axis = 11
Number of nodes on t axis = 48
Time step = 21.00 Minute
Bottom Slope = 0.000500000

Distance(Met.)	Initial Stage(Met.)
0.00	1.85
2000.00	1.85
4000.00	1.85
6000.00	1.85
8000.00	1.85
10000.00	1.85
12000.00	1.85
14000.00	1.85
16000.00	1.85
18000.00	1.85
20000.00	1.85

Lateral Flow = 0.00 Cumecs/Meter

EPSV = 0.005000
EPSV = 1.000000
THETA = 0.60
HYDROGRAPHS :

Station at 0.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.85	350.00
3	42.00	1.85	349.94
5	84.00	1.86	351.79
7	126.00	1.93	369.11
9	168.00	2.17	430.35
11	210.00	2.62	539.27
13	252.00	3.09	648.01
15	294.00	3.37	699.16
17	336.00	3.38	673.73
19	378.00	3.16	597.93
21	420.00	2.82	511.69
23	462.00	2.48	442.34
25	504.00	2.21	396.23
27	546.00	2.03	370.91
29	588.00	1.94	358.59
31	630.00	1.89	353.24
33	672.00	1.87	351.13
35	714.00	1.86	350.36
37	756.00	1.86	350.11
39	798.00	1.85	350.03
41	840.00	1.85	350.01

43	882.00	1.85	350.00
45	924.00	1.85	350.00
47	966.00	1.85	350.00

Station at 10000.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.85	350.00
3	42.00	1.85	349.49
5	84.00	1.85	349.79
7	126.00	1.86	351.12
9	168.00	1.89	360.71
11	210.00	2.04	396.81
13	252.00	2.36	474.28
15	294.00	2.78	572.28
17	336.00	3.12	643.53
19	378.00	3.26	659.17
21	420.00	3.20	624.41
23	462.00	3.00	563.30
25	504.00	2.73	499.08
27	546.00	2.46	445.12
29	588.00	2.24	405.88
31	630.00	2.08	380.39
33	672.00	1.97	365.43
35	714.00	1.92	357.39
37	756.00	1.88	353.38
39	798.00	1.87	351.50
41	840.00	1.86	350.65
43	882.00	1.86	350.27
45	924.00	1.85	350.11
47	966.00	1.85	350.05

Station at 20000.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.85	350.00
3	42.00	1.85	349.18
5	84.00	1.85	349.59
7	126.00	1.85	349.80
9	168.00	1.85	350.52
11	210.00	1.87	356.03
13	252.00	1.95	377.57
15	294.00	2.16	430.01
17	336.00	2.51	507.99
19	378.00	2.87	583.31
21	420.00	3.11	622.91
23	462.00	3.18	622.02
25	504.00	3.09	589.59
27	546.00	2.91	540.76
29	588.00	2.68	490.50
31	630.00	2.44	447.32
33	672.00	2.25	411.99
35	714.00	2.10	387.36
37	756.00	2.00	370.43
39	798.00	1.93	361.96
41	840.00	1.89	356.01
43	882.00	1.87	353.29
45	924.00	1.86	351.61
47	966.00	1.86	349.96

Average Velocity = 1.421 Met/Sec
Average Celerity = 4.656Met/Sec
Maximum number of iteration used = 9

STOP

APPENDIX-B

FEM PROGRAMME

MAIN PROGRAMME 'FEM'

```
C               This programme rout a flood using Finite Element Method.
C       Upstream boundary condition may be either a stage hydrograph or
C       a flow hydrograph. Downstream boundary condition may be either
C       kinematic or dynamic. Channel properties and inflow hydrographs
C       are to be called from predefined subroutines. Global matrix is
C       stored in banded form and LEQT1B, a library subroutine of IMSL
C       is used for their solution. This programme is developed at DEC
C       -1090 system at I.I.T. Kanpur. Programming language is FORTRAN-
C       10.

C       Input Data :
Card    <UNIT>
Card    < N,L,NQ,BC1,BC2,ALPHA,NS,DL >
Card    < X(1),INVT(1),Y(1,1) >
Card    <         .        .        >
Card    <         .        .        >
Card    <         .        .        >
Card    < X(I),INVT(I),Y(I,1) >
Card    <         .        .        >
Card    <         .        .        >
Card    <         .        .        >
Card    < X(N),INVT(N),Y(N,1) >
Card    < T(1),DT >
Card    < STN(1),...STN(I),...STN(NS) >
Card    < EPSY,EPSQ >
Card    < POSLAT(1),...POSLAT(I),...POSLAT(NQ) > if NQ is non zero.
Card    < QINT > if  BC1 is 1.

        where UNIT = 1 for MKS system of units
                   = 2 for FPS system of units
              N    =Number of nodes on X axis
              L    =Number of nodes on t axis
              BC1  = 1 if upstream stage hydrograph is given.
                   = 2 if upstream discharge hydrograph is given
              BC2  = 1 if rating curve is kinematic
                   = 2 if rating curve is dynamic
              NS   =Number of nodes where output is needed.
              DL   =Interval of time steps after which output is needed
              Y(I,J)=Flow depth at Ith node & Jth time level
              DT   =Time increament in minutes
             STN(I)=Ith node number at which output is needed
             QINT  =Initial discharge
             EPSY  =Permissible error in flow depth
             EPSQ  =Permissible error in discharge
             POSLAT(I)=Chainage of Ith tributary
             NQ    =Number of tributaries/lateral flows
             ALPHA =Time integeration factor(1.0-2.0)
             X(I)  =Chainage at Ith node
             INVT(I)=Invert R.L. at Ith node
             T(J)  =Time in minutes for Jth time level

        REAL MAN,MANAV,MAT,INHYD,INVT
        INTEGER COND,BC1,BC2,STN,DL,UNIT
        DIMENSION X(40),T(500),DXP(40),DXM(40),XL(320),STN(40)
        DIMENSION Q(40,500),Y(40,500),MAT(80,7),R(80),INVT(40)
        DIMENSION AR(40),V(40),ARP2(40),ARM2(40),ARP3(40),ARM3(40)
        DIMENSION ARP4(40),ARM4(40),VP3(40),VM3(40),QP3(40),QM3(40)
        DIMENSION DAYP2(40),DAYM2(40),DAYP4(40),DAYM4(40),FF(40)
        DIMENSION DAY(40),PR(40),MAN(40),MANAV(40),SF(40),D2AY(40)
        DIMENSION DYT(40),DQT(40),DPRY(4),QTRIB(30),UX(30),POSLAT(30)
        DIMENSION DYT1(40),DQT1(40),INHYD(500),SO(40),RY(40)
        DIMENSION QLAT(40),QUX(40)
        COMMON X,Y
```

```
501     FORMAT(5X,'Numbers of nodes on X axis =',I5,/,5X,'Number o
       1f nodes on t axis =',I5,/,5X,'Time step =',F6.2,' Minute')
502     FORMAT(/,5X,'Distance(Met.)',6X,'Invert Elevation(Met.)',
       15X,'Initial Stage(Met.)'/,(7X,F10.2,10X,F10.2,15X,F10.2))
602     FORMAT(/,5X,'Distance(Feet)',6X,'Invert Elevation(Feet)',
       15X,'Initial Stage(Feet)'/,(7X,F10.2,10X,F10.2,15X,F10.2))
603     FORMAT(/,5X,'S.N.',5X,'Position of tributary from origin(
       1Feet)',(/,6X,I2,8X,F12.2))
503     FORMAT(/,5X,'S.N.',5X,'Position of tributary from origin(
       1Meters)',(/,6X,I2,8X,F12.2))
504     FORMAT(/,5X,'EPSY =',F12.6,/,5X,'EPSQ =',F12.6)
505     FORMAT(10X,'FLOOD ROUTING BY FINITE ELEMENT METHOD - KINE
       1MATIC MODEL',/,10X,56('_'))
506     FORMAT(10X,'FLOOD ROUTING BY FINITE ELEMENT METHOD - DYNA
       1MIC MODEL',/,10X,54('_'))
507     FORMAT(/,5X,'Stage hydrograph is given at upstream')
508     FORMAT(/,5X,'Flow hydrograph is given at upstream')
512     FORMAT(/,5X,'Station at',F10.2,'  Meters')
612     FORMAT(/,5X,'Station at',F10.2,'  Feet')
513     FORMAT(5X,'S.N.',5X,'Time(Minute)',5X,'Stage(Met.)',5X
       1,'Discharge(Cumecs)')
613     FORMAT(5X,'S.N.',5X,'Time(Minute)',5X,'Stage(Feet)',5X
       1,'Discharge(Cusecs)')
514     FORMAT(5X,I4,5X,F8.2,10X,F6.2,10X,F12.2)
516     FORMAT(5X,'HYDROGRAPHS :')
517     FORMAT(5X,'NOT CONVERGING')
518     FORMAT(5X,'ALPHA =',F5.2)
519     FORMAT(5X,'Average Velocity =',F8.3,' Met/Sec',/,
       15X,'Average Celerity =',F8.3,'Met/Sec')
619     FORMAT(5X,'Average Velocity =',F8.3,' Feet/Sec',/,
       15X,'Average Celerity =',F8.3,'Feet/Sec')
520     FORMAT(5X,'Maximum number of iteration used =',I3)
        READ*,UNIT
        IF(UNIT.EQ.1)GR=9.81
        IF(UNIT.EQ.2)GR=32
        READ*,N,L,NQ,BC1,BC2,ALPHA,NS,DL
        READ*,((X(I),INVT(I),Y(I,1)),I=1,N)
        READ*,T(1),DT
        READ*,(STN(I),I=1,NS)
        READ*,EPSY,EPSQ
        IF(NQ.NE.0)READ*,(POSLAT(I),I=1,NQ)
        IF(BC2.EQ.1)TYPE 505
        IF(BC2.EQ.2)TYPE 506
        IF(BC1.EQ.1)TYPE 507
        IF(BC1.EQ.2)TYPE 508
        TYPE 501,N,L,DT
        IF(UNIT.EQ.1)TYPE 502,((X(I),INVT(I),Y(I,1)),I=1,N)
        IF(UNIT.EQ.2)TYPE 602,((X(I),INVT(I),Y(I,1)),I=1,N)
        IF(UNIT.EQ.1..AND.NQ.NE.0)TYPE 503,(POSLAT(I),I=1,NQ)
        IF(UNIT.EQ.2..AND.NQ.NE.0)TYPE 603,(POSLAT(I),I=1,NQ)
        TYPE 504,EPSY,EPSQ
        TYPE 518,ALPHA
        J=1
        K=0
        MIT=0
        INHYD(1)=HYDR(T(1))
        IF(BC1.EQ.1)GO TO 5
        QINT=INHYD(1)
        GO TO 6
5       READ*,QINT
6       AR(1)=AREA(1,1)
        V(1)=QINT/AR(1)
        AVV=V(1)
        CEL=SQRT(GR*Y(1,1))
        Q(1,1)=QINT
        DYT(1)=0
        DQT(1)=0
        DO 1 I=2,N
        LOCAL=I
        AR(I)=AREA(LOCAL,J)
```

```
      Q(I,J)=QINT
      V(I)=Q(I,J)/AR(I)
      AVV=AVV+V(I)
      CEL=CEL+SQRT(GR*Y(I,J))
      DYT(I)=0
      DQT(I)=0
      DXP(I-1)=X(I)-X(I-1)
1     SO(I-1)=(INVT(I-1)-INVT(I))/DXP(I-1)
      DO 2 I=2,L
      T(I)=T(I-1)+DT
      LOCAL=I
      INHYD(I)=HYDR(T(LOCAL))
      IF(BC1.EQ.1)Y(1,I)=INHYD(I)
      IF(BC1.EQ.2)Q(1,I)=INHYD(I)
2     CONTINUE
      TYPE 516
      DT=DT*60
3     J=J+1
      IF(NQ.EQ.0)GOTO 32
      CALL LATFLW(NQ,J,QTRIB,UX)
      DO 31 I=1,N-1
      QLAT(I)=0
      QUX(I)=0
      DO 31 II=1,NQ
      IF(POSLAT(II).LE.X(I).OR.POSLAT(II).GE.X(I+1))GO TO 30
      QLAT(I)=QLAT(I)+QTRIB(II)
      QUX(I)=QUX(I)+QTRIB(II)*UX(II)
30    IF(POSLAT(II).NE.X(I).AND.POSLAT(II).NE.X(I+1))GO TO 31
      QLAT(I)=QLAT(I)+QTRIB(II)/2
      QUX(I)=QUX(I)+QTRIB(II)*UX(II)/2
31    CONTINUE
32    DYT1(1)=DYT(1)
      DQT1(1)=DQT(1)
      Y(1,J)=Y(1,J-1)
      DO 4 I=2,N
      DYT1(I)=DYT(I)
      DQT1(I)=DQT(I)
      Y(I,J)=Y(I,J-1)
4     Q(I,J)=Q(I,J-1)
11    DO 12 I=1,N
      LOCAL=I
      AR(I)=AREA(LOCAL,J)
      V(I)=Q(I,J)/AR(I)
      DAY(I)=BREATH(LOCAL,J)
      D2AY(I)=DBY(LOCAL,J)
      PR(I)=PARAM(LOCAL,J)
      DPRY(I)=DPY(LOCAL,J)
      MAN(I)=FRICT(LOCAL,J)
      RY(I)=AR(I)/PR(I)
      SF(I)=MAN(I)**2*V(I)*ABS(V(I))/RY(I)**(4./3.)
      FF(I)=SF(I)/MAN(I)**2
      DYT(I)=ALPHA/DT*(Y(I,J)-Y(I,J-1))+(1-ALPHA)*DYT1(I)
      DQT(I)=ALPHA/DT*(Q(I,J)-Q(I,J-1))+(1-ALPHA)*DQT1(I)
12    CONTINUE
      DO 21 I=1,N-1
      DXM(I+1)=DXP(I)
      ARP2(I)=AR(I)+AR(I+1)
      ARM2(I+1)=ARP2(I)
      ARP3(I)=ARP2(I)+AR(I)
      ARM3(I+1)=ARM2(I+1)+AR(I+1)
      ARP4(I)=ARP3(I)+AR(I)
      ARM4(I+1)=ARM3(I+1)+AR(I+1)
      VP3(I)=V(I)*2+V(I+1)
      VM3(I+1)=V(I)+V(I+1)*2
      QP3(I)=Q(I,J)*2+Q(I+1,J)
      QM3(I+1)=Q(I,J)+Q(I+1,J)*2
      DAYP2(I)=DAY(I)+DAY(I+1)
      DAYM2(I+1)=DAYP2(I)
      DAYP4(I)=DAY(I)*3+DAY(I+1)
      DAYM4(I+1)=DAY(I)+DAY(I+1)*3
```

```
      MANAV(I)=(MAN(I)+MAN(I+1))/2
21    CONTINUE
      DO 61 I=2,N-1
      F1=DXM(I)*DOT(I-1)+2*(DXM(I)+DXP(I))*DOT(I)+DXP(I)*DOT(I+1)
     1+VM3(I)*(Q(I,J)-Q(I-1,J))+VP3(I)*(Q(I+1,J)-Q(I,J))
     2+QM3(I)*(V(I)-V(I-1))+QP3(I)*(V(I+1)-V(I))
     3-GR*(DXM(I)*ARM3(I)*SO(I-1)+DXP(I)*ARP3(I)*SO(I))
     4+0.5*GR*(DXM(I)*ARM2(I)*MANAV(I-1)**2*FF(I-1)
     5+(DXM(I)*ARM4(I)*MANAV(I-1)**2+DXP(I)*ARP4(I)*MANAV(I)**2)
     6*FF(I)
     7+DXP(I)*ARP2(I)*MANAV(I)**2*FF(I+1))
     8-3*(QUX(I-1)+QUX(I))
     9+GR*(ARM3(I)*(Y(I,J)-Y(I-1,J))+ARP3(I)*(Y(I+1,J)-Y(I,J)))
      F2=DXM(I)*DAYM2(I)*DYT(I-1)+DXP(I)*DAYP2(I)*DYT(I+1)
     1+(DXM(I)*DAYM4(I)+DXP(I)*DAYP4(I))*DYT(I)
     2+6*(Q(I+1,J)-Q(I-1,J))-6*(QLAT(I-1)+QLAT(I))
      II=I*2-1
      MAT(II,1)=0
      MAT(II,2)=-V(I-1)*(Q(I,J)-Q(I-1,J)-QM3(I))*DAY(I-1)/AR(I-1)
     1-GR*ARM3(I)+GR*DAY(I-1)*(Y(I,J)-Y(I-1,J))
     2-GR*(DXM(I)*DAY(I-1)*SO(I-1))
     3-0.5*GR*DXM(I)*ARM2(I)*MANAV(I-1)**2*2/3*FF(I-1)/AR(I-1)
     4*(5*DAY(I-1)-2*RY(I-1)*DPRY(I-1))
     5+0.5*GR*DXM(I)*MANAV(I-1)**2*(FF(I-1)+FF(I))*DAY(I-1)
      MAT(II,3)=DXM(I)*ALPHA/DT-VM3(I)
     1+(Q(I,J)-Q(I-1,J)-QM3(I))/AR(I-1)+V(I)-V(I-1)
     2+0.5*GR*DXM(I)*ARM2(I)*MANAV(I-1)**2*2*FF(I-1)/Q(I-1,J)
      MAT(II,4)=(2*(Q(I-1,J)-Q(I+1,J))
     1+(QP3(I)-QM3(I)))*V(I)*DAY(I)/AR(I)-GR*(ARM3(I)-ARP3(I))
     2+2*DAY(I)*(Y(I+1,J)-Y(I-1,J))
     3-GR*(DXM(I)*SO(I-1)+DXP(I)*SO(I))*2*DAY(I)
     4+0.5*GR*(DAY(I)*(DXM(I)*MANAV(I-1)**2*(FF(I-1)+3*FF(I))
     5+DXP(I)*MANAV(I)**2*(FF(I+1)+3*FF(I)))
     6-(DXM(I)*ARM4(I)*MANAV(I-1)**2
     7+DXP(I)*ARP4(I)*MANAV(I)**2)*2/3/AR(I)*(5*DAY(I)
     8-2*RY(I)*DPRY(I))*FF(I))
      MAT(II,5)=2*(DXM(I)+DXP(I))*ALPHA/DT+VM3(I)-VP3(I)
     1+2*(Q(I,J)-Q(I-1,J))/AR(I)+2*(Q(I+1,J)-Q(I,J))/AR(I)
     2+2*(-V(I-1)+V(I+1))+(QM3(I)-QP3(I))/AR(I)
     3+0.5*GR*(DXM(I)*ARM4(I)*MANAV(I-1)**2+DXP(I)*ARM4(I)
     4*MANAV(I)**2)*2*FF(I)/Q(I,J)
      MAT(II,6)=DAY(I+1)/AR(I+1)*(-V(I+1)*(Q(I+1,J)-Q(I,J))
     1-QP3(I)*V(I+1))
     2+GR*(ARP3(I)+DAY(I+1)*((Y(I+1,J)-Y(I,J))-DXP(I)*SO(I)))
     3+0.5*GR*DXP(I)*MANAV(I)**2*(FF(I)+FF(I+1))*DAY(I+1)
     4-0.5*GR*DXP(I)*ARP2(I)*MANAV(I)**2*2/3*FF(I+1)/AR(I+1)
     5*(5*DAY(I+1)-2*RY(I+1)*DPRY(I+1))
      MAT(II,7)=DXP(I)*ALPHA/DT+VP3(I)+(Q(I+1,J)-Q(I,J))/AR(I+1)
     1+QP3(I)/AR(I+1)+V(I+1)-V(I)+0.5*GR*DXP(I)*ARP2(I)*MANAV(I)**2
     2*2*FF(I+1)/Q(I+1,J)
      R(II)=-F1
      II=II+1
      MAT(II,1)=DXM(I)*DAYM2(I)*ALPHA/DT
     1+DXM(I)*D2AY(I-1)*(DYT(I-1)+DYT(I))
      MAT(II,2)=-6
      MAT(II,3)=DXM(I)*D2AY(I)*DYT(I-1)
     1+(DXM(I)*DAYM4(I)+DXP(I)*DAYP4(I))*ALPHA/DT
     2+3*(DXM(I)+DXP(I))*D2AY(I)*DYT(I)+DXP(I)*D2AY(I)*DYT(I+1)
      MAT(II,4)=0
      MAT(II,5)=DXP(I)*DAYP2(I)*ALPHA/DT
     1+DXP(I)*DYT(I+1)*D2AY(I+1)+DXP(I)*D2AY(I+1)*DYT(I)
      MAT(II,6)=6
      MAT(II,7)=0
      R(II)=-F2
61    CONTINUE
      MAT(2,1)=0
      MAT(2,2)=0
      MAT(2,3)=DXP(1)*(DAYP4(1)*ALPHA/DT+D2AY(1)
     1*(3*DYT(1)+DYT(2)))
      MAT(2,4)=-6
```

```
      MAT(2,5)=DXP(1)*(D2AY(2)*(DYT(1)+DYT(2))+DAYP2(1)*ALPHA/DT)
      MAT(2,6)=6
      MAT(2,7)=0
      F2=DXP(1)*DAYP4(1)*DYT(1)+DXP(1)*DAYP2(1)*DYT(2)
     1+5*(Q(2,J)-Q(1,J)-QLAT(1))
      R(2)=-F2
      II=N*2-1
      MAT(II,1)=0
      MAT(II,2)=DXM(N)*((DYT(N-1)+DYT(N))*D2AY(N-1)+DAYM2(N)*ALPHA
     1/DT)
      MAT(II,3)=-6
      MAT(II,4)=DXM(N)*(D2AY(N)*(3*DYT(N)+DYT(N-1))+DAYM4(N)*ALPHA
     1/DT)
      MAT(II,5)=6
      MAT(II,6)=0
      MAT(II,7)=0
      F2=DXM(N)*DAYM2(N)*DYT(N-1)+DXM(N)*DAYM4(N)*DYT(N)
     1+6*(Q(N,J)-Q(N-1,J)-QLAT(N-1))
      R(II)=-F2
      MAT(1,1)=0
      MAT(1,2)=0
      MAT(1,3)=0
      MAT(1,4)=0
      MAT(1,5)=1
      MAT(1,6)=0
      MAT(1,7)=0
      R(1)=0
      IF(BC1.EQ.2)GO TO 13
      MAT(1,4)=1
      MAT(1,5)=0
13    NN=N*2
      QN=1/MAN(N)*AR(N)*RY(N)**(2./3.)*SQRT(SO(N-1))
      IF(BC2.EQ.1)GO TO 66
      QFAC=(1-1/SO(N-1)*(Y(N,J)-Y(N-1,J))/DXM(N)
     1-V(N)/SO(N-1)/GR*(V(N)-V(N-1))/DXM(N)
     2-1/SO(N-1)/GR*(DQT(N)-V(N)*DAY(N)*DYT(N))/AR(N))
      QFAC=SQRT(QFAC)
56    MAT(NN,1)=-QN*0.5/QFAC*(1-V(N)/GR*V(N-1)/AR(N-1)
     1*DAY(N-1))/SO(N-1)/DXM(N-1)
      MAT(NN,2)=-QN*0.5/QFAC*V(N)/SO(N-1)/GR/AR(N-1)/DXM(N)
      MAT(NN,3)=-QN*0.5/QFAC*(-1/SO(N-1)
     1/DXM(N)+V(N)/SO(N-1)/GR
     2/AR(N)*(2*V(N)-V(N-1))/DXM(N)*DAY(N))+1/MAN(N)*QFAC*SO(N-1)
     3**0.5*(2/3*RY(N)**(5./3.)*DPRY(N)-RY(N)**
     4(2./3.)*5/3*DAY(N))+1/SO(N-1)/GR*(DQT(N)-V(N)*DAY(N)
     5*DYT(N))/AR(N)**2*DAY(N)-1/SO(N-1)/GR*(DQT(N)-V(N)*DAY(N)
     6*ALPHA/DT-V(N)*D2AY(N)*DYT(N)+V(N)/AR(N)*DAY(N)**2
     7*DYT(N))/AR(N)
      MAT(NN,4)=1-QN*0.5/QFAC*(-1/SO(N-1)/GR/AR(N)/DXM(N)
     1*(2*V(N)-V(N-1)))-1/SO(N-1)/GR*(ALPHA/DT-DAY(N)
     2*DYT(N)/AR(N))/AR(N)
      R(NN)=-Q(N,J)+QN*QFAC
      GO TO 67
66    MAT(NN,1)=0
      MAT(NN,2)=0
      MAT(NN,3)=1/MAN(N)*SO(N-1)
     2**0.5*(2/3*RY(N)**(5./3.)*DPRY(N)-RY(N)**
     1(2./3.)*5/3*DAY(N))
      MAT(NN,4)=1
      R(NN)=-Q(N,J)+QN
67    MAT(NN,5)=0
      MAT(NN,6)=0
      MAT(NN,7)=0
      CALL LEQT1B(MAT,NN,3,3,80,R,1,80,0,XL,IER)
      COND=0
      DO 71 I=1,N
      LOCAL=I*2
      Y(I,J)=Y(I,J)+R(LOCAL-1)
      Q(I,J)=Q(I,J)+R(LOCAL)
      IF(ABS(R(LOCAL-1)).GT.EPSY.OR.ABS(R(LOCAL)).GT.EPSQ)COND=1
```

```
71      CONTINUE
        K=K+1
        IF(MIT.LT.K)MIT=K
        IF(K.EQ.40)GO TO 95
        IF(COND.EQ.1)GO TO 11
        K=0
        DO 81 I=1,N
        AVV=AVV+V(I)
81      CEL=CEL+SQRT(GR*Y(I,J))
        IF(J.GE.L)GO TO 3
        AVV=AVV/(N*L)
        CEL=CEL/(N*L)
        DO 91 I=1,NS
        IF(UNIT.EQ.1)TYPE 512,X(STN(I))
        IF(UNIT.EQ.2)TYPE 612,X(STN(I))
        IF(UNIT.EQ.1)TYPE 513
        IF(UNIT.EQ.2)TYPE 613
91      TYPE 514,((J,T(J),Y(STN(I),J),Q(STN(I),J)),J=1,L,DL)
        IF(UNIT.EQ.1)TYPE 519,AVV,CEL
        IF(UNIT.EQ.2)TYPE 619,AVV,CEL
        TYPE 520,MIT
        STOP
95      TYPE 517
        STOP
        END
```

```
                EXAMPLE-I
       FUNCTIONS/SUBROUTINES '1E'
       FUNCTION AREA(I,J)
       COMMON X,Y
       DIMENSION X(40),Y(40,500)
       AREA=6.1*Y(I,J)
       RETURN
       END
       FUNCTION BREATH(I,J)
       COMMON X,Y
       DIMENSION X(40),Y(40,500)
       BREATH=6.1
       RETURN
       END
       FUNCTION DBY(I,J)
       COMMON X,Y
       DIMENSION X(40),Y(40,500)
       DBY=0
       RETURN
       END
       FUNCTION PARAM(I,J)
       COMMON X,Y
       DIMENSION X(40),Y(40,500)
       PARAM=6.1+2*Y(I,J)
       RETURN
       END
       FUNCTION DPY(I,J)
       COMMON X,Y
       DIMENSION X(40),Y(40,500)
       DPY=2
       RETURN
       END
       FUNCTION FRICT(I,J)
       COMMON X,Y
       DIMENSION X(40),Y(40,500)
       FRICT=0.02
       RETURN
       END
       SUBROUTINE LATFLW(NQ,J,QTRIB,UX)
       DIMENSION QTRIB(30),UX(30)
       DO 1 I=1,NQ
       QTRIB(I)=200
1      UX(I)=0
       RETURN
       END
       FUNCTION HYDR(I)
       IF(I.GT.20)GO TO 1
       HYDR=23.34+1.083*I
       RETURN
1      IF(I.GE.60)GO TO 2
       HYDR=23.34+(60-I)*0.8415
       RETURN
2      HYDR=23.34
       RETURN
       END
```

INPUT DATA(KINEMATIC) '1F1.CDR' :

```
1
5 16 0 2 1 1.5 3 1
0 4.8 1.83
800 3.6 1.83
1600 2.4 1.83
2400 1.2 1.83
3200 0 1.83
0 5
1 3 5
0.001 10
```

OUTPUT '1E1.DAT' :

FLOOD ROUTING BY FINITE ELEMENT METHOD - KINEMATIC MODEL

Flow hydrograph is given at upstream
numbers of nodes on X axis = 5
number of nodes on t axis = 16
Time step = 5.00 Minute

Distance(Met.)	Invert Elevation(Met.)	Initial Stage(Met.)
0.00	4.80	1.83
800.00	3.60	1.83
1600.00	2.40	1.83
2400.00	1.20	1.83
3200.00	0.00	1.83

EPSY = 0.001000
EPSO = 10.000000
ALPHA = 1.50
HYDROGRAPHS :

Station at 0.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.83	23.34
2	5.00	2.08	31.75
3	10.00	2.46	40.17
4	15.00	2.83	48.59
5	20.00	3.21	57.00
6	25.00	3.26	52.79
7	30.00	3.13	48.59
8	35.00	2.98	44.38
9	40.00	2.81	40.17
10	45.00	2.61	35.96
11	50.00	2.41	31.75
12	55.00	2.19	27.55
13	60.00	1.97	23.34
14	65.00	1.86	23.34
15	70.00	1.84	23.34
16	75.00	1.82	23.34

Station at 1600.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.83	23.34
2	5.00	1.87	24.82
3	10.00	2.04	29.34
4	15.00	2.35	36.67
5	20.00	2.71	44.97
6	25.00	3.00	50.51
7	30.00	3.10	50.81
8	35.00	3.07	48.27
9	40.00	2.98	45.19
10	45.00	2.85	41.74
11	50.00	2.69	38.05
12	55.00	2.50	34.25
13	60.00	2.31	30.38
14	65.00	2.12	27.11
15	70.00	1.99	25.15
16	75.00	1.91	24.28

Station at 3200.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.83	23.34
2	5.00	1.84	23.74
3	10.00	1.90	24.85
4	15.00	2.06	28.00
5	20.00	2.36	33.65
6	25.00	2.70	40.59
7	30.00	2.97	46.02
8	35.00	3.07	48.20
9	40.00	3.05	47.68
10	45.00	2.95	45.74
11	50.00	2.82	43.04
12	55.00	2.67	39.86
13	60.00	2.50	36.41
14	65.00	2.32	32.89
15	70.00	2.15	29.73
16	75.00	2.03	27.30

Average Velocity = 2.359 Met/Sec
Average Celerity = 4.885Met/Sec
Maximum number of iteration used = 4
STOP

OUTPUT '1E2.DAT' :

FLOOD ROUTING BY FINITE ELEMENT METHOD - DYNAMIC MODEL

Flow hydrograph is given at upstream
Numbers of nodes on X axis = 5
Number of nodes on t axis = 16
Time step = 5.00 Minute

Distance(Met.)	Invert Elevation(Met.)	Initial Stage(Met.)
0.00	4.80	1.83
800.00	3.60	1.83
1600.00	2.40	1.83
2400.00	1.20	1.83
3200.00	0.00	1.83

EPSY = 0.001000
EPSQ = 10.000000
ALPHA = 1.50
HYDROGRAPHS :

Station at 0.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.83	23.34
2	5.00	2.08	31.75
3	10.00	2.46	40.17
4	15.00	2.85	48.59
5	20.00	3.23	57.00
6	25.00	3.29	52.79
7	30.00	3.16	48.59
8	35.00	3.00	44.38
9	40.00	2.81	40.17
10	45.00	2.61	35.96
11	50.00	2.39	31.75
12	55.00	2.17	27.55
13	60.00	1.94	23.34
14	65.00	1.84	23.34
15	70.00	1.81	23.34
16	75.00	1.80	23.34

Station at 1600.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.83	23.34
2	5.00	1.87	24.83
3	10.00	2.05	29.39
4	15.00	2.36	36.76
5	20.00	2.72	45.05
6	25.00	3.01	50.52
7	30.00	3.11	50.74
8	35.00	3.08	48.19
9	40.00	2.98	45.14
10	45.00	2.84	41.70
11	50.00	2.68	38.01
12	55.00	2.50	34.21
13	60.00	2.30	30.35
14	65.00	2.11	27.10
15	70.00	1.98	25.16
16	75.00	1.91	24.31

Station at 3200.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.83	23.34
2	5.00	1.83	23.76
3	10.00	1.88	25.06
4	15.00	2.01	28.37
5	20.00	2.25	34.14
6	25.00	2.56	40.98
7	30.00	2.83	46.04
8	35.00	2.98	47.79
9	40.00	3.02	47.09
10	45.00	2.97	45.16
11	50.00	2.88	42.56
12	55.00	2.75	39.51
13	60.00	2.60	36.18
14	65.00	2.43	32.83
15	70.00	2.26	29.83
16	75.00	2.11	27.53

Average Velocity = 2.359 Met/Sec
Average Celerity = 4.885Met/Sec
Maximum number of iteration used = 6
STOP

EXAMPLE-II
SUBROUTINE/FUNCITONS '2E' :

```
FUNCTION AREA(I,J)
DIMENSION Y(40),Y(40,500)
COMMON X,Y
AREA=0.0008*Y(I,J)**4+2*Y(I,J)**3-10*Y(I,J)**2+100*Y(I,J)+100
RETURN
END
FUNCTION BREATH(I,J)
DIMENSION X(40),Y(40,500)
COMMON X,Y
BREATH=0.0032*Y(I,J)**3+6*Y(I,J)**2-10
1*Y(I,J)+100
RETURN
END
FUNCTION PARAM(I,J)
DIMENSION X(40),Y(40,500)
COMMON X,Y
PARAM=0.0016*Y(I,J)**3+4.0*Y(I,J)**2-15*Y(I,J)+
1160
RETURN
END
FUNCTION FRICT(I,J)
DIMENSION X(40),Y(40,500)
COMMON X,Y
FRICT=0.025
RETURN
END
FUNCTION DBY(I,J)
DIMENSION X(40),Y(40,500)
COMMON X,Y
DBY=0.0096*Y(I,J)**2+12*
1Y(I,J)-10
RETURN
END
FUNCTION DPY(I,J)
DIMENSION X(40),Y(40,500)
COMMON X,Y
DPY=0.004*Y(I,J)**2+6.4*
1Y(I,J)-15
RETURN
END
FUNCTION HYDR(T)
DATA QB/350/,QP/700/,TP/300/,GAMA/1.15/
HYDR=QB+(QP-QB)*(T/TP)**(1./(GAMA-1.)/EXP(GAMA-1.)*(1.-T/TP))
RETURN
END
SUBROUTINE LATFLW(NQ,J,QTRIB,UX)
DIMENSION QTRIB(30),UX(30)
RETURN
END
```

INPUT '2E1.CDR' :

```
1
11 41 0 2 1 2 3 2
0000 100 1.85
2000 99 1.85
4000 98 1.85
6000 97 1.85
8000 96 1.85
10000 95 1.85
12000 94 1.85
14000 93 1.85
16000 92 1.85
18000 91 1.85
20000 90 1.85
0 25
1 6 11
0.005 2.0
```

OUTPUT '2d1.DAT' :

FLOOD ROUTING BY FINITE ELEMENT METHOD - KINEMATIC MODEL

Flow hydrograph is given at upstream
Numbers of nodes on X axis = 11
Number of nodes on t axis = 41
Time step = 25.00 Minute

Distance(Met.)	Invert Elevation(Met.)	Initial Stage(Met.)
0.00	100.00	1.85
2000.00	99.00	1.85
4000.00	98.00	1.85
6000.00	97.00	1.85
8000.00	96.00	1.85
10000.00	95.00	1.85
12000.00	94.00	1.85
14000.00	93.00	1.85
16000.00	92.00	1.85
18000.00	91.00	1.85
20000.00	90.00	1.85

EPSY = 0.005000
EPSQ = 2.000000
ALPHA = 2.00
HYDROGRAPHS :

Station at 0.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.85	350.00
3	50.00	1.85	350.07
5	100.00	1.87	355.23
7	150.00	2.02	397.91
9	200.00	2.46	511.16
11	250.00	3.02	644.00
13	300.00	3.36	700.00
15	350.00	3.32	652.03
17	400.00	3.01	551.88
19	450.00	2.60	459.36
21	500.00	2.26	399.59
23	550.00	2.04	369.29
25	600.00	1.93	356.56
27	650.00	1.88	351.98
29	700.00	1.87	350.54
31	750.00	1.86	350.13
33	800.00	1.86	350.03
35	850.00	1.85	350.01
37	900.00	1.85	350.00
39	950.00	1.85	350.00
41	1000.00	1.85	350.00

Station at 10000.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.85	350.00
3	50.00	1.85	349.37
5	100.00	1.85	349.65
7	150.00	1.87	353.55
9	200.00	1.96	376.77
11	250.00	2.25	449.72
13	300.00	2.71	558.55
15	350.00	3.07	636.47
17	400.00	3.20	642.13
19	450.00	3.09	591.30
21	500.00	2.83	520.41
23	550.00	2.53	456.75
25	600.00	2.27	409.31
27	650.00	2.06	380.06
29	700.00	1.97	364.06
31	750.00	1.91	355.84
33	800.00	1.88	352.61
35	850.00	1.86	350.96
37	900.00	1.86	350.27
39	950.00	1.85	350.57
41	1000.00	1.85	350.53

Station at 20000.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.85	350.00
3	50.00	1.85	349.35
5	100.00	1.85	349.35
7	150.00	1.85	349.21
9	200.00	1.86	351.82
11	250.00	1.93	364.67
13	300.00	2.16	408.16
15	350.00	2.56	488.26
17	400.00	2.93	570.46
19	450.00	3.12	612.07
21	500.00	3.08	604.30
23	550.00	2.89	560.74
25	600.00	2.64	505.25
27	650.00	2.39	453.15
29	700.00	2.18	412.11
31	750.00	2.04	384.82
33	800.00	1.95	367.71
35	850.00	1.90	358.53
37	900.00	1.88	353.84
39	950.00	1.87	352.34
41	1000.00	1.86	351.48

Average Velocity = 1.420 Met/Sec
Average Celerity = 4.649Met/Sec
Maximum number of iteration used = 17

STOP

INPUT DATA(DYNAMIC) '2E2.CDR' :

```
1
11 41 0 2 2 2 3 2
0000 100 1.85
2000 99 1.85
4000 98 1.85
6000 97 1.85
8000 96 1.85
10000 95 1.85
12000 94 1.85
14000 93 1.85
16000 92 1.85
18000 91 1.85
20000 90 1.85
0 25
1 6 11
0.005,2.0
```

FLOOD ROUTING BY FINITE ELEMENT METHOD - DYNAMIC MODEL

```
Flow hydrograph is given at upstream
Numbers of nodes on X axis =   11
Number of nodes on t axis =   41
Time step = 25.00 Minute
```

Distance(Met.)	Invert Elevation(Met.)	Initial Stage(Met.)
0.00	100.00	1.85
2000.00	99.00	1.85
4000.00	98.00	1.85
6000.00	97.00	1.85
8000.00	96.00	1.85
10000.00	95.00	1.85
12000.00	94.00	1.85
14000.00	93.00	1.85
16000.00	92.00	1.85
18000.00	91.00	1.85
20000.00	90.00	1.85

```
EPSY =     0.005000
EPSQ =     2.000000
ALPHA = 2.00
HYDROGRAPHS :
```

Station at 0.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.85	350.00
3	50.00	1.85	350.07
5	100.00	1.87	355.23
7	150.00	2.02	397.91
9	200.00	2.46	511.16
11	250.00	3.02	644.00
13	300.00	3.36	700.00
15	350.00	3.33	652.03
17	400.00	3.02	551.88
19	450.00	2.61	459.36
21	500.00	2.26	399.59
23	550.00	2.04	369.29
25	600.00	1.92	356.56
27	650.00	1.88	351.98
29	700.00	1.86	350.54
31	750.00	1.85	350.13
33	800.00	1.85	350.03
35	850.00	1.85	350.01
37	900.00	1.85	350.00
39	950.00	1.85	350.00
41	1000.00	1.85	350.00

Station at 10000.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.85	350.00
3	50.00	1.85	349.37
5	100.00	1.85	349.65
7	150.00	1.87	353.55
9	200.00	1.96	376.76
11	250.00	2.25	449.65
13	300.00	2.70	557.80
15	350.00	3.07	635.93
17	400.00	3.20	642.48
19	450.00	3.08	592.22
21	500.00	2.83	521.03
23	550.00	2.53	456.25
25	600.00	2.27	409.30
27	650.00	2.06	380.02
29	700.00	1.97	363.21
31	750.00	1.91	355.58
33	800.00	1.88	352.26
35	850.00	1.86	350.93
37	900.00	1.86	350.65
39	950.00	1.85	350.85
41	1000.00	1.85	350.58

Station at 20000.00 Meters

S.N.	Time(Minute)	Stage(Met.)	Discharge(Cumecs)
1	0.00	1.85	350.00
3	50.00	1.85	349.35
5	100.00	1.85	349.34
7	150.00	1.85	349.53
9	200.00	1.86	352.34
11	250.00	1.91	366.71
13	300.00	2.09	411.76
15	350.00	2.43	492.90
17	400.00	2.81	570.82
19	450.00	3.03	607.59
21	500.00	3.08	596.93
23	550.00	2.95	555.41
25	600.00	2.73	502.49
27	650.00	2.49	453.21
29	700.00	2.27	414.32
31	750.00	2.10	386.96
33	800.00	1.99	369.69
35	850.00	1.92	359.50
37	900.00	1.90	354.50
39	950.00	1.88	352.61
41	1000.00	1.88	351.95

Average Velocity = 1.420 Met/Sec
Average Celerity = 4.650Met/Sec
Maximum number of iteration used = 17
STOP